Ductile fracture modelling and \(J-Q\) fracture mechanics: a constraint based fracture assessment approach

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**ABSTRACT.** The reduced state of stress triaxiality observed in shallow cracked components allows an increased capacity to resist crack propagation compared to that observed in deeply cracked specimens. This may be regarded as a higher fracture toughness value which allows a reduction in the inherent conservatism when assessing components in low constraint conditions. This study uses a two-parameter fracture mechanics approach (\(J-Q\)) to quantify the level of constraint in a component (e.g. a pipe with a surface crack) and in fracture test specimens, i.e. single edge tension [SE(T)] and compact tension [C(T)] specimens, of varying constraint level. The level of constraint of the component is matched to a specific test specimen and therefore the ability of the structure to resist fracture is given by the fracture toughness of the test specimen with a similar \(J-Q\) response. Fracture toughness values for different specimens have been obtained from tearing resistance curves (\(J-R\) curves) constructed by means of a virtual testing framework. The proposed engineering approach shows that the combination of a local approach and two-parameter fracture mechanics can be used as a platform to perform more accurate fracture assessments of defects in structures with reduced constraint conditions.

**KEYWORDS.** Crack tip constraint; Ductile fracture modelling; \(J-Q\) fracture mechanics.

**INTRODUCTION**

In structural integrity assessments, the fracture toughness value used to determine the onset of fracture, \(K_{\text{mat}}\), is commonly derived from deeply cracked specimens with almost square ligaments, using recommended testing standards and validity criteria (e.g. ASTM E1820 [1] and ESIS-P2 [2]). These are designed to ensure high constraint conditions near the crack tip that correspond to lower-bound toughness values independent of specimen size and geometry.

There exist cases in which low constraint conditions can be demonstrated. For example, in the Oil and Gas (O&G) industry, during installation, pipeline girth welds are predominantly loaded in tension even if the pipe is globally subjected to bending. The flaw sizes of interest are usually controlled by the weld pass height and are therefore relatively small,
typically 2-6 mm in height [3]. Both these aspects result in reduced crack tip constraint in the component compared to the deeply notched standard specimens.

Furthermore, there is experimental evidence showing that panels loaded in tension exhibit higher resistance to fracture since these conditions lead to lower constraint around the crack [4]. As a result, in these cases, the material capacity to withstand load is underestimated and it would be useful to perform assessments with a fracture resistance value obtained from a test specimen with a crack tip constraint condition similar to that in the actual component [5].

Materials can exhibit a change in toughness with specimen geometry for both cleavage and ductile fracture modes. Here, attention is focussed on ductile crack propagation behaviour (microvoid coalescence). A ductile fracture simulation approach has been implemented in previous work [6,7] to evaluate the fracture resistance curves (J–R) for different test specimens. Although these procedures are useful tools to evaluate fracture resistance for structural components, the development and calibration of the finite element assessment (FEA) model requires extensive expertise and the application of the procedures becomes prohibitive for routine assessments. An alternative framework is then of interest for more rapid assessments.

The J–Q two-parameter fracture mechanics [8,9] approach has been extensively used to characterize elastic–plastic crack front fields. The parameter Q characterizes the degree of crack tip constraint, by quantifying the level of deviation of stress/strain fields from reference fields.

In this work, we conduct investigations on the J–Q two parameter characterisation approach to compare the constraint conditions of a pipe under different loading modes with those observed in C(T) and SE(T) specimens. The aim of this is to support the use of a low constraint fracture toughness value by showing that at the same applied driving force (J), the level of constraint (Q) at the pipe is similar to that in the SE(T) specimen.

**THEORETICAL BACKGROUND**

**Two parameter J–Q theory**

In small-scale yielding, there is always a zone of single parameter (K, J, CTOD) dominance. The crack-tip conditions are fully defined by the single parameter, whose value depends on load, crack size and geometry. The situation changes as plasticity develops when the loss of constraint becomes apparent (e.g., fully plastic response or shallow cracks), and single parameter dominance does not hold. Under these circumstances, the stresses near the crack tip are not given by the single parameter but also depend on the configuration (loading type, geometry and material properties). In low constraint geometries the near tip stress distribution can be significantly lower than the high constraint J-dominant state.

The J–Q approach to elastic-plastic fracture mechanics was introduced to remove some of the conservatism inherent in the single parameter approach based on the J integral. The following equation provides an approximate description of the near tip stress field, over physically significant distances [8,9]:

\[ \sigma_{ij} = \sigma_{ij}^{\text{ref}} + Q \sigma_0 \delta_{ij} \]

where \( \delta_{ij} \) is the Kronecker delta, \( \sigma_0 \) is the yield stress and \( \sigma_{ij}^{\text{ref}} \) is a reference field, often taken as the HRR field, the near crack tip fields for power-law plastic materials derived in [10,11]. Thus, the Q-factor quantifies the difference between the actual local stress at a certain reference location near the crack tip and the theoretical HRR-stress field and is given by:

\[ Q = \frac{\sigma_{ij} - \sigma_{ij}^{\text{ref}}}{\sigma_0} \]

The actual stress field in a component and the HRR field in the forward sector of the crack-tip region differ by an approximately uniform hydrostatic stress independently of distance from the crack tip, for given values of J [8,9]. Therefore, eq (1) means that with the addition of the second parameter, a range of stress states can be obtained at a fixed deformation level (as characterised by J), differing by a hydrostatic stress (as characterised by Q). In practice, the stress field is more complex than eq. (1) but this simplification has been found to apply for the region at the crack tip where [8,9], corresponding to the near crack tip zone where the fracture process zone (FPZ) for both cleavage and ductile
fracture is active but outside the area where crack blunting becomes significant. Negative $Q$ values indicate lower constraint conditions compared to the reference field and positive $Q$ values higher constraint conditions.

A local approach to ductile fracture

An alternative framework for constraint analyses and effective fracture toughness assessment is the application of failure models, often referred to as local approaches. Local approaches couple the loading history (stress-strain) near the crack-tip region with micro-structural features of the fracture mechanisms involved [12]. Since the fracture event is described locally, the mechanical factors affecting fracture are included in the predictions of the model. The parameters depend only on the material and not on the geometry, and this leads to improved transferability from specimens to structures than one- and two-parameter fracture mechanics methods [13].

A fracture model accounting for the ductile damage processes has been used to quantify the increased resistance of blunt defects relative to sharp ones and to demonstrate that a loss of constraint leads to an increase in the fracture properties of these materials. A phenomenological model [14] based on a stress modified fracture strain concept was used in [6,7] to construct $J$-$R$ curves of notched compact tension $C(T)$ and single edge tension $S(E(T)$.

It has been demonstrated that true fracture strain for ductile materials is strongly dependent on the level of stress triaxiality [15-17]. The model used in this study therefore uses an exponential relationship between the true fracture strain, $\varepsilon_f$, and stress triaxiality:

$$\varepsilon_i = \alpha \exp\left[-\gamma \frac{\sigma_{\text{eq}}}{\sigma_i}\right] + \beta$$ (3)

where $\alpha$, $\beta$ and $\gamma$ are material constants obtained by fitting test data for smooth and notched bars and the triaxiality is:

$$\frac{\sigma_{\text{eq}}}{\sigma_i} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3\sigma_i}$$ (4)

where $\sigma_i (i=1-3)$ are principal stresses and $\sigma_i$ is the von Mises stress.

Using a FE analysis technique, this model is implemented in a step-by-step procedure in which at each loading step, the incremental damage, $\Delta \omega$, produced by incremental strain is assessed and added to the total damage, $\omega$, produced in previous steps. The quantification of the incremental damage definition is performed in each finite element of the model as follows:

$$\Delta \omega_i = \frac{\Delta \varepsilon^p_i}{\varepsilon_f} \ ; \ \omega_i = \omega_{i-1} + \Delta \omega_i$$ (5)

where $\Delta \varepsilon^p_i$ is the equivalent plastic strain increment and $\varepsilon_f$ is determined by the local triaxiality in the element using Eq. (3). When the total damage becomes equal to unity ($\omega=1$), local failure is assumed to occur at the element and the initiation and propagation of a crack is simulated by reducing all the stress components to a sufficiently small value to make the contribution of the element to the resistance of the component negligible.

It should be noted that this local approach with the simulation procedure briefly summarised above has been verified by comparison with experimental data on fracture toughness test specimens and pressurised pipes which serves as validation for this purpose. All material constants in Eq. (3) with the crack tip element size for the material and tensile properties used in this study were determined by the procedure and also verified with experimental data. More details on the numerical implementation of the model can be found in [6,7,18].

**Finite Element Analysis**

A shallow cracked $S(E(T)$ specimen and a deeply cracked $C(T)$ specimen were modelled by means of 3-D finite elements. The relevant dimensions of these specimens and the pipe component assessed in this work are shown in Fig. 1. The material properties used in the numerical models are for an API X65 steel used in [6,19].
Fig. 2 shows the FE models. Due to symmetric conditions of loading and geometry a quarter of C(T) and SE(T) specimens were modelled, to improve computational efficiency. One half of the pipe was modelled in order to be able to apply pure bending.

In [6], it was shown that element size in the defect section affects the results for the damage accumulation process; therefore, this value must be determined by comparison with experimental results. For API X65, the element size is 0.15mm [6]. The material constants to apply the fracture criterion of Eq. (3) for API X65, based on the defined element size are $\alpha=3.29$, $\beta=-1.54$ and $\gamma=0.01$. The damage model is implemented within C3D8 hexahedron solid elements using the ABAQUS UHARD and USDFLD user-defined subroutines [20] coded in FORTRAN 90. The total numbers of elements/nodes in the FE models are from 23,366/26,016, 82,520/88,929 and 117,595/130,416 for the SE(T) specimen, C(T) specimen and the pipe, respectively.

**Figure 1:** Schematic illustration of fracture toughness specimens showing the dimensions: (a) C(T) specimen; (b) SE(T) specimen; (c) Pipe with surface circumferential crack.

**Figure 2.** Finite element models: (a) C(T) specimen; (b) SE(T) specimen; (c) Pipe with surface circumferential crack.
RESULTS

The numerical \( J \)-\( R \) curves obtained by the implementation of the damage model are shown in Fig. 3. A standard deeply cracked C(T) specimen and a shallow cracked SE(T) specimen are modelled. The use of the ESIS P2 procedure [2] for the estimation of the effective initiation fracture toughness is illustrated. Next, the \( J \)-\( Q \) approach is applied to the pipe component for internal pressure and pure bending in order to show the effect of the loading mode on constraint level. In general, the value of \( Q \) depends on load magnitude (and therefore on \( J \)) as well as loading mode, being proportional to load in small-scale yielding but weakly dependent on \( J \) at large loads. The values of \( Q \) have therefore been evaluated at applied \( J \) values, see Fig. 4, for the pipe at loads which cover the range of \( J \) at initiation in C(T) and SE(T) specimens. It can be seen that at these values, the stress fields in the pipe when plotted against normalised distance are weakly dependent on \( J \). Hence, \( Q \) is also weakly dependent on \( J \) in this practical range. It can then be assumed that the pipe would have the same effective initiation toughness as a specimen with the same \( J \)-\( Q \) value. For fracture assessments where it is not possible to match the \( Q \) value of the pipe with that of a test specimen, the specimen with the closest higher value of \( Q \) will be a conservative choice.

The \( Q \)-stress is generally evaluated at the distance \( r = 2J/\sigma_0 \) from the crack tip and using the opening stress obtained by detailed finite element analysis, Eq. (2). The reference field in Eq. (1) is obtained from a boundary layer analysis at the same applied \( J \) with \( T=0 \), as this enables the approach to be applied to materials which do not follow the power-law form which enables the HRR field to be used as the reference field in Eq. (1).

CONCLUSIONS

Crack size, loading mode and material properties can have a strong effect on constraint conditions, affecting the material resistance to fracture. In this work, finite element ductile fracture simulation has been used to construct \( J \)-\( R \) curves for 2 test specimens with different constraint conditions. The ductile fracture model only considers a small area ahead of the crack tip.
(geometry independent) and couples the loading history (stress-strain) with phenomenological features of the microstructural fracture mechanism (material + loading history dependent). In addition, a Two Parameter Fracture Mechanics approach has been applied to match the constraint conditions present in a defective structural component to those present in the test specimens. By doing this, the $J-Q$ approach allows an improved assessment of the fracture resistance of the component, by using the fracture resistance of the test specimen with similar constraint conditions to reduce over-conservatism in fracture assessments.

![Normalized crack-opening stress distribution for the different components.](image)

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**REFERENCES**


