Fatigue methodology for life predictions for the wheel-rail contact area in large offshore turret bearings

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ABSTRACT. The present report presents a fatigue life prediction method for large roller bearings applied in the turret turn table for large loading buoy units. The contact points between wheel and rail in these bearings are subjected to a multi-axial fluctuating stress situation and both surface wear and fatigue cracking may occur. A methodology based on the Dang Van fatigue criterion is adopted. The criterion is based on an equivalent stress defined as a combination of the fluctuation of the shear stress from its mean value at a critical plane and the associated hydrostatic stress at the given time. The present work is supporting the theoretical model by extensive laboratory testing. Both full scale testing of wheel on rail and small scale testing for characterizing the steel material are carried out. An experimental program was carried out with the high strength stainless steel S165M. The Dang Van stress concept is applied in combination with the Random Fatigue Limit Method (RFLM) for life data analyses. This approach gives the opportunity to include both finite lives and the run-outs in a rational manner without any presumption of the existence of a fatigue limit in advance of the data. This gives a non-linear S-N curve for a log-log scale in the very high cycle regime close to the fatigue limit. It is demonstrated how the scatter in fatigue limit decreases when the Dang Van stress concept is applied and that the fatigue limit is occurring beyond 10⁷ cycles.

KEYWORDS. Dang Van criterion; Random Fatigue Limit Method; Rolling Contact Fatigue.
INTRODUCTION

Offshore loading buoy units are often constructed with a large central turret in order to allow the floating unit to weathervane. The geostationary moored turret has a turn table with a bearing arrangement in the top. The turn table can be based on a sliding bearing concept, but for larger turrets a roller bearing design is usually preferred. The wheels are rolling on a circular rail mounted on the top of the floaters deck structure, see Fig. 1. The durability of these bearings during a typical target service life of 25 years is a matter of concern. During inspection of rails in service, surface cracks have been detected on former installations. These cracks may eventually obstruct the rotation that allows the floating unit to weathervane. The cracks are thus considered as a hazard due to the risk of encountering unforeseen loading condition if the buoy should be locked in one direction. Furthermore, the bearings are so huge that replacement in-situ will be very cumbersome and expensive. This makes the fatigue Safe Life Limit (SLL) a major design criterion. Further details are given in [1].

The objectives of the present work are:
• Study the fatigue resistance of the Martensitic-Austenitic stainless steel S165M particularly when subjected to multiaxial stress situation typical for Rolling Contact Fatigue (RCF).
• Apply the Random Fatigue Limit Method (RFLM) as a supplement to conventional case methods that in the authors’ opinion are outdated.
• Demonstrate the ability of the Dang Van multi-axial stress concept to reduce scatter in the fatigue limit. This is demonstrated by small scale testing with specimens subjected to pulsating and alternating tension.
• The Dang Van based fatigue limit can be used to ensure safe life condition during the service for the offshore installation in question. This is verified by full scale fatigue testing.

The practical design based on the obtained model is verified by full scale testing and the reader is referred to [2] on this subject.

THE DANG VAN STRESS CONCEPT FOR MULTIAXIAL FATIGUE

The sub-surface stress situation in the contact point between wheel and rail is multi-axial and may lead to fatigue cracking under repetitive loading. It is a common hypothesis that it is the shear stress amplitude that is the key variable to the fatigue crack initiation, but also other stress contributions may play a role. The multi-axial fatigue criterion according to the Dang Van approach reads, [3, 5]:

$$\max_{t} \left[ \tau_{a}(t) + a_{DL} \sigma_{h}(t) \right] < \tau_{e}$$

(1)

where $\tau_{a}$ is the acting shear stress amplitude (deviation from mean shear stress) at any time, whereas $\sigma_{h}$ is the simultaneous acting hydrostatic stress. $\tau_{e}$ is the shear stress amplitude strength (fatigue endurance limit) when no other stress
components are present. The hydrostatic stress is a simplified bulk measure for any complex stress situation. The material constants $\tau_e$ and $a_{DV}$ are determined from two test series. The curve associated with Eq. (1) is shown in Fig. 2. The parameter $\tau_e$ is the ordinate value of the curve whereas $a_{DV}$ is the slope of the curve. The two test series will give data at the two points that are denoted pulsating (P) and alternating (A) stress in Fig. 2. The line is typically defined based on the assumption that the fatigue limit is defined at $N=10^7$ cycles. It is not clarified in the original work how scatter is treated, [3, 5]. It is one of the goals of the present work to bring some light on these issues.

![Figure 2: The Dang Van design line for the fatigue limit at N=10⁷ cycles.](image)

The rail will be the most critical component with respect to fatigue and a martensitic stainless steel 165M is selected. This is to have a steel that has high resistance both with respect to fatigue and corrosion as the bearing is located in the splash zone. The steel has a typical yield stress of 700 MPa. For S165M there is not much data available on the fatigue strength in the literature. The acting shear stress amplitude $\tau_a$ in a typical wheel-rail design will typically be 140 MPa at zero hydrostatic stress for the extreme load case. Hence, based on preliminary assumptions taken from the literature studies the stress situation shall not lead to fatigue cracking. This will be corroborated by the testing described in the next section.

**THE FATIGUE TEST SERIES**

Two special small scale test series were carried out to characterize the multiaxial fatigue resistance. The first test series is under pulsating tension, whereas the second one is under alternating tension. Both test series are planned with classical dog-boned specimens with diameter 10 mm. The surface finish was $R=0.8$ that reflects the surface of the rail in service. The small scale test results were used as theoretical support for the large scale rolling test. A number of 10 specimens were manufactured for each test series, i.e. 20 tests in total. The test series were carried out as follows:

*Test series 1: Pulsating tension, $R=0.05$ at 100 Hz*

The actual stress situation is shown in Fig. 3 where the x-axis is the specimen pulling direction. As can be seen the plane oriented 45 degrees will be subject to the maximum shear stress $\tau$ and a normal stress component $\sigma$. It is this stress combination that will be the driving force for the crack initiation.

![Figure 3: Stress situation in the material for a rod subjected to tension in x-direction.](image)
**Test series 2: Alternating compression and tensile testing at R=-1**

For this series the actual stress situation will again be as shown in Fig. 3, but as this stress history is partly compressive the maximum hydrostatic stress decreases compared with test series 1.

The following equations for the stress situation apply for the two test series:

\[
\begin{align*}
\sigma_b &= \frac{\Delta \sigma}{3}, \quad \tau_s = \frac{\Delta \sigma}{4} & \text{Test series 1} \\
\sigma_b &= \frac{\Delta \sigma}{2 \Delta^3}, \quad \tau_s = \frac{\Delta \sigma}{4} & \text{Test series 2}
\end{align*}
\]

Due to the lower hydrostatic stress in test series 1 this series will sustain higher shear amplitude as predicted by Eq. (1) and illustrated in Fig. 2. All the tests were run to failure or stopped at \(N=2 \cdot 10^7\) cycles. Failure is defined by total fracture of the specimen. The fatigue limit is defined at \(N=10^7\) cycles as is in accordance with rule and regulations. Typically applied normal stress for test series 1 was close to a range of 450 MPa, whereas the stress range for series 2 was close to 500 MPa. The applied loading was tuned in after the first tests have been completed. The RFLM model was applied to determine the fatigue limit, see description next section. Hence, the two test series gives us two points on the Dang Van design curve as shown in Fig. 2. Consequently, the slope of the curve denoted \(a_{DV}\) is determined.

**The random fatigue limit methodology for data analysis**

The conventional statistical analyses of data points are based on linear regression of fatigue life data only, [5] or by the staircase method for the fatigue limit only, [6]. In the present case with large scatter in both fatigue life and the fatigue limit these methods are regarded as less appropriate.

Due to the uncertainty and large scatter in fatigue life in the region close to \(10^7\) cycles, an S-N curve based on a Random Fatigue-Limit Model (RFLM) is adopted in the present work, [4]. The basic feature of the model is that both fatigue life and the fatigue-limit are treated as random variables simultaneously. The fatigue limit should not be treated separately as it is done for the bi-linear curves. The S-N curve obtained from the RFLM will not have an abrupt change from an inclined straight line to a horizontal line, but a gradually change in slope as stress ranges get very low. It then remains to be seen if the slope goes asymptotically towards a horizontal line as the number of cycles increases. If this is not the case the existence of a fatigue-limit should be rejected. We shall not elaborate the method in the present article but outline the most important characteristics. The method applies a Maximum Likelihood Method that gives a rational treatment of run outs. The basic equations are:

\[
\ln(N) = \beta_0 - \beta_0 \ln(\Delta \gamma) + \epsilon
\]

where \(\ln\) denotes the natural logarithm and \(\gamma = \Delta \gamma_0\) is the fatigue-limit. The parameters \(\beta_0\) and \(\beta_1\) are fatigue curve coefficients. For given sample data \(w_i\) and \(x_i\) from various test specimens \(i = 1, \ldots, n\), the model parameters can be determined by the Maximum Likelihood (ML) function:

\[
L(Q) = \prod_{i=1}^{n} \left[ f_x(w_i;\cdot,Q) \right]^{\delta_i} \left[ 1 - F_x(w_i;\cdot,Q) \right]^{1-\delta_i}
\]

where \(\delta_i = 1\) if \(w_i\) is a failure and \(\delta_i = 0\) if \(w_i\) is a censored observation (run out). The vector \(Q\) contains the model parameters:

\[
Q = \left( \beta_0, \beta_1, \sigma, \mu_\gamma, \tau_\gamma \right)
\]

where \(\sigma\) is the standard deviation for the natural logarithm to fatigue life, whereas \(\mu_\gamma\) and \(\tau_\gamma\) are the mean value and standard deviation respectively for the fatigue limit \(\ln(\gamma)\). Once these parameters have been determined from optimization
of Eq. (5), the corresponding confidence intervals can be obtained by a profile likelihood method using the profile ratio of the variables together with chi-square statistics. When the parameters are determined we can calculate the fatigue life for a chosen probability $p$ of failure using Eq. (4). Hence, the median curve and percentile curves for design purpose are obtained. For further details the reader is referred to Pascal and Meeker, [4]. It should be mentioned that the optimization of Eq. (5) may be difficult due to local optimum points.

**TEST RESULTS AND DISCUSSION**

The obtained life data were analyzed in 3 steps based on the methodology described in the section above. The RFLM approach was first used to determine the mean fatigue limit for the direct applied stress range $\Delta \sigma$ for test series 1 and 2 separately. The results are shown in the lower and upper part of Fig. 4. The fatigue limit was determined for both test series. As can be seen in the mean value for the fatigue limit for the series 1 is 457 MPa where it reaches 535 MPa for test series 2 when taken $10^7$ cycles. This increase from test series 1 to series 2 is explained by the fact that series 2 has a smaller maximum normal stress on the 45 degrees plane than series 1 has, see Fig. 4. Hence, the shear stress amplitude in series 1 will be more damaging than the shear amplitude of the same magnitude in series 2 due to the fact that the simultaneously occurring normal stress $\sigma$ has decreased for series 2. This phenomenon is taken into account by the hydrostatic stress term in Eq. (1). A similar analysis carried out by the staircase method gave typically 5% higher mean fatigue limits, [2]. The large scatter for series 2 is peculiar. As can be seen the data points for the rupture have a positive correlation for cycles versus stress range. More data points are needed to get the expected behavior.

Subsequently, in the second step, the associated Dang Van equivalent stress limit was determined for the two series based on Eqs. (1). This also gives the necessary information to determine the Dang van constants. The results are shown in Fig. 2 given in Tab. 1 based on the mean stresses and the Dang Van constant is determined to 0.28. This slope is obtained by entering the results from Eq. (2) and (3) respectively into Eq. (1). The obtained value is somewhat lower than obtained for ferritic steels. In the last third step all the data was gathered on one plot applying the Dang Van equivalent stress as the explaining variable to fatigue life. This equivalent stress is defined:

$$\sigma_{eq} = \max_i \left[ r_i(t) + a_{PV} \sigma_i \right]$$

(7)

The advantage of this analysis is that the number of data points is doubled and the confidence intervals for the parameters become smaller. The data points and the associated curves are shown in Fig. 5. As can be seen the results for the Dang
Van equivalent stress fatigue limit is 140 MPa at a 2.5% percentile defined at N=10⁷ cycles. As can be seen the curve has not become asymptotically horizontal at N=10⁷ cycles, this will occur when approaching 10⁸ cycles. In addition to the mean design curve based on the fatigue limit as was shown in Fig. 2, an entire S-N curve is now obtained as shown in Fig. 5. The purpose of the curve is to be able to calculate the damage accumulation according to the Miner summation for variable amplitude loading. The curve is directly applicable in the test region, i.e. at Dang Van stress ranges below 200 MPa. The shape of the curve will result in exclusion of many stress cycles in a typical in-service load spectrum as they will become non-damaging according to the curve. Very many of these cycles will typically be below a range of 160 MPa. Tab. 1 gives the scatter band for the fatigue limit when defined between 10⁷ and 2∙10⁷ cycles. The standard deviation $\sigma_{\gamma}$ is also given. As can be seen the magnitude is 0.11 and 0.22 for series 1 and 2 respectively when each series is treated separately. Applying the Dang Van equivalent amplitude stress for all data gathered gives a relative scatter band close to 0.16 which is in between the values obtained when each series is handled separately. It demonstrates how the influence of R ratio implicitly is taken care of by the Dang Van stress multiaxial stress concept.

<table>
<thead>
<tr>
<th>Data points</th>
<th>Explaining stress</th>
<th>Maximum scatter band</th>
<th>Standard deviation $\sigma_{\gamma}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>All R=0 and R=-1</td>
<td>$\Delta\sigma_e$</td>
<td>180</td>
<td>-</td>
</tr>
<tr>
<td>R=0 only</td>
<td>$\Delta\sigma_e$</td>
<td>110</td>
<td>0.11</td>
</tr>
<tr>
<td>R=-1 only</td>
<td>$\Delta\sigma_e$</td>
<td>80</td>
<td>0.22</td>
</tr>
<tr>
<td>All R=0 and R=-1</td>
<td>$\sigma_{\text{eq}}$ (Eq. (7))</td>
<td>35</td>
<td>0.16</td>
</tr>
</tbody>
</table>

Table 1: Table Scatter in stress data between N=10⁷ and 2∙10⁷ cycles dependent on explaining stress.

As a close to this section it shall briefly be mentioned that the practical application of the established S-N curve directly obtained by first require that the maximum acting equivalent stress in the rail during service shall be less than the curve in Figure 5. The acting stress can be found by empirical equations or by advanced Finite Element Models for the contact between wheel and rail. We will not pursue this procedure in the present article; the reader is referenced to [1, 2].

**CONCLUSIONS**

The Dang Van defined equivalent stress is suggested for fatigue life predictions under the multi-axial stress situations imposed at the wheel rail contact point. Originally the method was applied for verifying the fatigue limit usually chosen at 10⁷ cycles. In the present work a Random Fatigue Limit Model is applied for a more consistent statistical data analysis. The result is a non-linear S-N curve for a log-log scale in the high cycle regime close to the fatigue
limit. The fatigue limit is defined as the horizontal asymptote is appearing at a given number of cycles. It is demonstrated how difference in the fatigue limit for different R ratios reduces when applying the Dang Van stress concept. The fatigue resistance of the Martensitic-Austenitic stainless steel S165M is quite good, but somewhat poorer than for comparable non-stainless steels with the same yield stress. There is also a considerable scatter in the fatigue life limit for the present steel. The applicability of the results is verified by full scale testing and a practical multiaxial fatigue design tool has been established and applied in practice. This will be presented in a future work.

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REFERENCES