Multiaxial fatigue assessment of crankshafts by local stress and critical plane approach

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ABSTRACT. For multiaxially-loaded parts several stress-based fatigue assessment concepts are applicable mostly taking uniaxial test results as basis. These approaches work well in case of proportional loading states, but on contrary, for non-proportional stress conditions, implying a change of the principal stress direction, deviations in the fatigue life estimation may occur. The aim of this study is to evaluate the cyclic multiaxial material behavior experimentally and to proof the applicability of stress-based methods to assess the fatigue strength.

The investigated base materials incorporate the commonly applied crankshaft steels 50CrMo4 and 34CrNiMo6 without surface-layer post-treatments. Extensive fatigue tests with small-scale specimens are performed to evaluate the material behavior under cyclic loading. The experiments include basic uniaxial characterizations, such as notch stress sensitivity and effect of loading type, including tests under tension, rotating bending, and torsion loading. Additionally, combined loading tests with proportional and non-proportional situations are presented to reveal the fatigue resistance for multiaxial stress states. Significant loading conditions, such as proportional stress under rotating bending and torsion, and further on, non-proportional effects like phase shifts and varying frequency ratios are presented. The local fatigue strength assessment is performed on the basis of the critical plane approach, where the normal and shear stresses are transformed in numerous cutting planes. Equivalent stress hypotheses are applied and compared with the
experiments showing that the Huber-Mises-Hencky criterion fits well to the test results in case of proportional rotating bending and torsion loading.

**KEYWORDS.** Multiaxial fatigue assessment; Local stress concept; Critical plane approach; Crankshaft.

**INTRODUCTION**

This paper presents a multiaxial fatigue strength assessment of crankshafts based on different local stress approaches. Focus of the application is laid on stationary gas engines exhibiting a high level of electrical and thermal efficiency. By the permanent increasing demand to optimize specific power output, a reliable and accurate numerical fatigue analysis is of utmost importance. The investigations are separated in a uniaxial and multiaxial section, mainly focusing on the commonly applied steel 50CrMo4. The results are partially supplemented and compared to a previous work [1] incorporating extensive tests with the steel types 34CrNiMo6 and 42CrMo4.

**UNIAXIAL FATIGUE STRENGTH CHARACTERIZATION**

In this chapter a basic characterization of the base material focusing on 50CrMo4 is provided. Emphasis is laid on the major fatigue-related influences for a notch stress-based fatigue assessment, in particular
- notch stress sensitivity, and
- type of loading.

**Notch sensitivity**

The effect of various notched specimens on the fatigue strength of the crankshaft base material 50CrMo4 is presented in Fig. 1. The results reveal a significant decrease by about 50% of the high-cycle fatigue limit at five million load-cycles in case of the most sharply notched (notch radius $R=0.8\ \text{mm}$) compared to the unnotched specimen under tension/compression (T/C) loading. All tests are performed at a stress ratio of $R=-1$.

![Figure 1. Influence of notch and loading type (Tension/Compression and Rotating Bending) on uniaxial fatigue strength (50CrMo4).](image)
For commonly applied steel materials, the reduction in fatigue strength due to notches is less than the elastic theory by terms of the stress concentration factor $K_t$ would estimate. Notch influences are generally specified by the fatigue notch factor $K_f$ representing the ratio of the unnotched to the notched fatigue resistance. A study in [2] provides an overview and verification of several models evaluating $K_f$ for different base materials. Engineering feasible concepts are mostly based on the relative stress gradient $\chi'$ due to an efficient automated evaluation of finite element results. One method among them is presented by [3] calculating the fatigue support effect $n=K_t/K_f$ as follows:

$$n = 1 + \left( \frac{\sigma_{B,R}}{\sigma_{T/C,R}} - 1 \right) \left( \frac{\chi'}{2/d} \right)^{K_D}$$

(1)

Thereby, $\sigma_{B,R}$ equals the fatigue strength under bending and $\sigma_{T/C,R}$ the fatigue strength under tension/compression loading, $d$ is the diameter of the specimens by which $\sigma_{B,R}$ is evaluated, and $K_D$ is a material-dependent parameter. The application of the model in order to assess the local notch fatigue strength is depicted in Fig. 2 showing a good accordance to the fatigue test results.

In addition to the local endurance limit, a method to estimate the slope $k$ and the transition knee point $N_D$ of the local S/N-curve is presented in [3]. Thereby, the determination is performed on the basis of the evaluated support effect and an exponential interpolation among a minimum and maximum value $k_{\text{min}}$ and $k_{\text{max}}$, and $N_D_{\text{min}}$ and $N_D_{\text{max}}$. These values are statistically evaluated by numerous fatigue tests, whereby $k_{\text{min}}$ and $N_D_{\text{min}}$ represent a technically highest-notched case, and $k_{\text{max}}$ and $N_D_{\text{max}}$ the unnotched condition.

**Normal vs. shear stress loading**

As the crankshafts local loading scenario is not only affected by normal stresses, additionally the fatigue strength under shear stress (torsion) loading is investigated. The results in Fig. 3 highlight a distinctive difference, whereas the shear stress high-cycle fatigue strength $\tau_{T,R}$ features only 51% compared to the behaviour under normal stress.

**MULTIAXIAL FATIGUE STRENGTH CHARACTERIZATION**

**Proportional loading**

An overview of different concepts to assess the fatigue strength under proportional loading is given in [4]. One common approach is introduced by Gough and Pollard [5] taking the normal and shear stress by an elliptical correlation into account, see Eq. (2).
Multiaxial fatigue tests under proportional rotating bending and torsion loading are carried out in order to assess the fatigue behavior under multiaxial loading for the investigated steel 50CrMo4. The tests are performed in the unnotched specimen condition at a loading stress ratio of $R=-1$ and a shear to normal stress ratio of $\frac{\tau}{\sigma_b}=0.5$. Fig. 4 summarizes the uniaxial results under pure normal and pure shear stress, and the combined proportional loading mode. The application of the model according to Gough and Pollard illustrates a good accordance to the test results. Moreover, the method demonstrates its applicability for numerous test results incorporating steel, cast iron and aluminium specimens in [6].

![Figure 4. Multiaxial fatigue behaviour under proportional loading (50CrMo4).](image)

**CRITICAL PLANE APPROACH**

Among stress, strain and energy based multiaxial fatigue criteria, the critical plane approach is one widespread method due to its effectiveness and broad application range. The general procedure of the approach is a reduction of the multiaxial stress condition to an equivalent uniaxial stress, firstly introduced by [7]. Definition of the cutting plane angle $\phi$ and calculation of the associated normal and shear stress is shown in Fig. 5.

![Figure 5. Evaluation of normal and shear stress in critical plane.](image)

A review of critical plane orientations in multiaxial fatigue failure criteria for metallic materials is provided in [8]. One basic equivalent stress concept is introduced by Huber-Mises-Hencky assuming a ratio of normal to shear stress fatigue strength of $\sqrt{3}$, see Eq. 3 in case of plain stress condition. Another approach by Lamé takes only the portion of the (maximum) normal stress into account, see Eq. 4.

$$\sigma_{eq} = \sqrt{\frac{\sigma_n^2}{2} + 3 \tau_n^2}$$  \hspace{1cm} (3)
\[ \sigma_{v2} = \sigma_n \] (4)

Fig. 6 presents the stress-time distribution for the three tested load-cases and the corresponding characteristic of the equivalent stress amplitude by Huber-Mises-Hencky depending on the actual load time \( t \) and cutting plane angle \( \varphi \). The values are normalized to the experimentally evaluated bending fatigue strength \( \sigma_{B,R} \) in accordance to Fig. 4.

On the basis of the two presented equivalent stress criteria, the maximum allowable normal (bending \( \sigma_n = \sigma_{xx} \)) and shear (torsion \( \tau_T = \tau_{xy} \)) stress amplitudes are determined as summarized in Tab. 1. In case of only bending loading \( (\tau_T / \sigma_B = 0) \), \( \sigma_{v1} \) reveals an almost comparable bending fatigue strength \( \sigma_B \) as the experiments indicate, whereas the calculated value is slightly conservative. An application of the maximum normal stress criteria \( \sigma_{v2} \) also agrees well to the test results. For the combined loading state \( (\tau_T / \sigma_B = 0.5) \) both concepts lead to a slightly overestimation of the corresponding fatigue strength values. Torsion loading without bending \( (\tau_T / \sigma_B = \infty) \) again overrates the experimental values, but however, in case of \( \sigma_{v1} \) by just 7% compared to 49% on the basis of \( \sigma_{v2} \). The accordant critical plane angles \( \varphi \) also differ significantly, whereas for \( \sigma_{v1} \) the values match acceptably well to the failure modes of the tested specimens.
Normalized stress

<table>
<thead>
<tr>
<th>$\tau_T/\sigma_B$</th>
<th>$\sigma_B=0.94$ &amp; $\tau_T=0.00$</th>
<th>$\sigma_B=1.00$ &amp; $\tau_T=0.00$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($\varphi=30^\circ/150^\circ$)</td>
<td>($\varphi=0^\circ/180^\circ$)</td>
</tr>
<tr>
<td>$\tau_T/\sigma_B=0.5$</td>
<td>$\sigma_B=0.74$ &amp; $\tau_T=0.37$</td>
<td>$\sigma_B=0.84$ &amp; $\tau_T=0.42$</td>
</tr>
<tr>
<td></td>
<td>($\varphi=55^\circ/170^\circ$)</td>
<td>($\varphi=25^\circ$)</td>
</tr>
<tr>
<td>$\tau_T/\sigma_B=\infty$</td>
<td>$\sigma_B=0.00$ &amp; $\tau_T=0.58$</td>
<td>$\sigma_B=0.00$ &amp; $\tau_T=1.00$</td>
</tr>
<tr>
<td></td>
<td>($\varphi=0^\circ/90^\circ$)</td>
<td>($\varphi=45^\circ/135^\circ$)</td>
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Table 1. Multiaxial fatigue assessment for 50CrMo4

**Fatigue Assessment of Crankshafts**

By the aid of a multi-body simulation the developing time-dependent loading situation of a crankshaft for a 16-cylinder gas engine is evaluated. A subsequent numerical analysis of one selected notch illustrates the complex non-proportional stress condition considering the local normal and shear stress at the surface, see Fig. 7.

![Figure 7. Normal and shear stress distribution over one firing sequence (rotation of crankshaft by 720°) at one selected notch of a 16-cylinder gas engine.](image)

**Effect of Phase Shift and Frequency Ratio**

As the local stress distribution over one firing sequence shows a distinctive non-proportional load-case, the effect of phase shift and frequency ratio for normal and shear stress loading is experimentally investigated in [1] for the base material 34CrNiMo6. The results reveal that a phase shift of $45^\circ (\pi/4)$ leads to an increase of 3 % and a phase shift of $90^\circ (\pi/2)$ of 6 % compared to the fatigue strength of proportional normal and shear stress loading. Furthermore, a frequency ratio of $f_\tau/f_\sigma=2$ indicates just a minor decrease by 1 %, but a ratio of $f_\tau/f_\sigma=3$ shows a more relevant reduction by 6 %. In addition to the analysis of the material behaviour, the resultant time- and cutting plane-dependent equivalent stress is of utmost importance to assess the local fatigue strength properly. Hence, further investigations incorporating equivalent stress analysis with various basic non-proportional load-scenarios and the presented load distribution for a 16-cylinder gas engine is scheduled. Additional multiaxial stress criteria, such as integral-based methods [9], are applied in order to proof their accuracy.

**Summary**

Uniaxial fatigue tests are performed to characterize basic influences, such as notch stress sensitivity and loading type, of the base material 50CrMo4. The results show, firstly, that the stress gradient model introduced by [3] is well applicable, and secondly, that a significant difference between normal and shear stress loading is evaluated
exhibiting a ratio $\sigma_R/\tau_R$ of 1.96. Proportional multiaxial fatigue tests illustrate that the elliptical model by Gough and Pollard [5] matches well to the results. An application of the Huber-Mises-Hencky equivalent stress criterion shows, in comparison to the maximum normal stress concept, good accordance to the experiments. The effect of non-proportional loading on the fatigue strength of 34CrNiMo6 specimens is presented based on experimental investigations in [1]. Further extensive analyses incorporating the Huber-Mises-Hencky and additional multiaxial criteria, e.g. integral-based approaches [9], are scheduled.

REFERENCES