ABSTRACT. In this paper we investigated the influence of consecutive dynamic and gigacycle fatigue loads on the lifetime of aluminum-magnesium alloy AlMg6. Preloading of samples was achieved during dynamic tensile tests in the split-Hopkinson bar device. Fatigue tests were conducted on Shimadzu USF-2000 ultrasonic fatigue testing machine. This machine provides $10^9$-$10^{10}$ loading cycles with the amplitude from 1 to several dozens of microns and frequency of 20 kHz, which reduces dramatically the testing time in the comparison to the classical fatigue testing machines. The New-View 5010 interferometer–profiler of high structural resolution (resolution of 0.1 nm) was used for qualitative fracture surface analysis, which provided the data allowing us to find correlation between mechanical properties and scale-invariant characteristics of damage induced roughness formed under dynamic and gigacycle fatigue loading conditions.

Original form of the kinetic equation was proposed, which links the rate of the fatigue crack growth and the stress intensity factor using the scale invariant parameters of fracture surface roughness. The scale invariance characterizes the correlated behavior of multiscale damage provides the link of crack growth kinetics and the power exponent of the modified Paris law.

KEYWORDS. Fracture; Gigacycle fatigue; Scaling; Surface morphology; Fractal analysis; Paris law.

INTRODUCTION

The assessment of the lifetime of critical engineering structures, in particular those for aircraft engines, poses qualitatively new fundamental problems related to evaluation of the reliability of materials under cyclic loading in excess of $10^9$-$10^{10}$ cycles corresponding to the so-called gigacycle fatigue range. This interest is caused by the fact that the fatigue lifetime of critical structures operating under cyclic loading conditions exceeds a gigacycle fatigue range. The gigacycle fatigue range can be characterized by some features, where of special interest is the range pertaining to the number of cycles $N\approx10^9$. The behavior of materials in this range reveals some qualitative changes in the mechanisms governing both the nucleation of cracks and their propagation.

The influence of random statistic and dynamic loads on the lifetime of materials under gigacycle fatigue regime is a subject of much current interest for aircraft motor companies in the context of solving the problem of reliability (longevity) of materials under real operating conditions [1]. For instance, that concerns to the lifetime estimation of gas turbine engine blades during their collision with solid particles usually called foreign object damage. The solution to this problem needs
In this paper, aluminum-magnesium alloy (AlMg6) samples (Fig. 1) are tested.

**Kinetic Equation for Fatigue-Crack Growth**

The universal character of kinetic law establishing a relationship between the growth rate \( \frac{dl}{dN} \) of a fatigue cracks and a change in the stress intensity coefficient \( \Delta K \) has been extensively studied both experimentally and theoretically. The power laws originally established by Paris [2] (and presently referred to as the Paris law) reflect the self-similar nature of fatigue crack kinetics. This law is related to a nonlinear character of damage evolution in the vicinity of the crack tip (called the “process zone”):

\[
\frac{dl}{dN} = A(\Delta K)^m
\]

where \( A \) and \( m \) are the experimentally determined constants. For a broad class of materials and wide range of crack propagation velocities under high cycle fatigue conditions, the exponent is typically close to \( m = 2-4 \).

The self-similar aspects of the fatigue crack growth were studied by Barenblatt, Ritchie [3,4] using the assumption concerning intermediate self-similarity of fatigue crack kinetics to introduce the following variables for the representation of the crack growth rate \( a = \frac{dl}{dN} \) (where \( l \) is the crack length and \( N \) is the number of cycles): \( a_1 = \Delta K \) is the stress intensity factor; \( a_2 = E \) is the Young modulus; \( a_3 = l_{sc} \) is the scale related to the correlated behavior in the ensemble of defects on the scale \( a_4 = L_{pz} \) associated with the process zone. 3D New View roughness data within the crack process zone (Fig.2) supported the existence of mentioned characteristic scales: the scale of process zone \( L_{pz} \) and correlation length \( l_{sc} \) that is the scale when correlated behavior of defect induced roughness has started.

Using the \( \Pi \)-theorem and taking into account the dimensions of variables \( [dl/dN] = L, [\Delta K] = FL^{-3/2}, [l_{sc}] = [L_{pz}] = L, \) and \( [E] = FL^{-2} \), the kinetic equation for the crack growth:

\[
\Phi(l_{sc}, l_{pz}) = \frac{dl}{dN}
\]

can be written as:

\[
\frac{dl}{dN} = \Phi \left( \frac{\Delta K}{E \sqrt{l}}, \frac{L_{pz}}{l_{sc}} \right)
\]

Estimation of the values \( \Delta K(l_{sc}) \ll 1 \) and \( L_{pz} / l_{sc} \gg 1 \) allowed one to suggest an intermediate-asymptotic character of the crack growth kinetics for Eq. (3) in the following form:

\[
\frac{dl}{dN} = \left( \frac{\Delta K}{E \sqrt{l}} \right)^a \left( \frac{L_{pz}}{l_{sc}} \right)^b
\]

where \( l = l / l_{sc} \). Introducing the parameter \( C = \left( \frac{L_{pz}}{l_{sc}} \right)^b \), we can reduce the scaling relation (4) to the following form analogous to the Paris law:

\[
\frac{dl}{dN} = C \left( \frac{\Delta K}{E \sqrt{l}} \right)^a
\]

\[423\]
where \( \alpha \) is a universal exponent. This form is similar to the equation proposed by Hertzberg for \( l_{bc} \rightarrow b \), where \( b \) is the Burgers vector.

**MATERIALS AND EXPERIMENTAL CONDITIONS**

Dynamic preloading of aluminum-magnesium alloy (AlMg6) samples (Fig. 1) was realized using the split Hopkinson pressure bar set-up at the strain rates of \( \sim 10^3 \) s\(^{-1} \), after which the samples were subject to cyclic loading at room temperature. Then the fractography of fracture surface pattern was studied using the roughness data by interferometer–profiler New-View 5010.

Fatigue tests were carried out using the resonance type of testing machine of (Shimadzu USF-2000) that provides the cyclic loading using the generator transforming the frequency of 50 Hz into ultrasonic electrical sinusoidal signal of frequency 20 kHz by piezoelectric transducer, generating longitudinal ultrasonic waves at a frequency 20 kHz and the mechanical stress with maximum amplitude in the gauge length (mid-section) of the sample. Deviation of the frequency by 0.5 kHz was associated with the damage critical stage and considered as failure precursor related to the initiation of a crack with characteristic size of \( \sim 2 \) mm. The level of applied stresses allowed us to investigate fatigue life up to the values associated with \( 10^{10} \) cycles. The fatigue scenario was studied for the stress amplitude 105, 120, 130 MPa corresponding to a critical number of cycles of about \( \sim 10^{10} \) estimated for initially unstressed materials subject to gigacycle fatigue tests.

![Figure 1: Sample geometry (sizes are given in millimeters).](image-url)

The roughness of fracture surfaces was measured by the high-resolution New-View 5010 interferometer-profiler (providing x2000 magnification) and then was analyzed using the assumption concerning fractal geometry of fracture surface profile associated with correlated behavior of multiscale defect structures on the scale of process zone \( L_{pz} \) that preceded to the crack growth.

The fatigue tests provided the fractured samples of two types. The samples of the first type were broken during the fatigue test. The samples of the second type revealed pronounced variations in the resonance frequency associated with fatigue crack origin. The fracture surfaces of the samples of the first and second type were uncovered by cooling the samples with liquid nitrogen and breaking them in the minimal cross-section of samples. It is assumed that the fatigue fracture surface of the samples subjected to gigacycle loading has already been formed during fatigue tests and occupies the largest part of the fracture surface, which is determined by a change in the resonance frequency.

The crack origin in cylindrical samples undergoing high cycle loading in the range \( 10^6-10^7 \) is started from the surface (Fig. 2a). For the samples subject to cyclic loads in the range exceeding \( 10^6 \) cycles the crack formation is started in the bulk of the sample. The fracture surface in this case exhibits a fatigue zone known as a “fish-eye” which is a particular feature of such fatigue regimes. The central part of this region comprises a fracture nucleus surrounded by the region of refined (submicrocrystalline) structure (region 2 in Fig. 2b).

The New-View scanning was realized over the zones of fatigue crack growth (Fig. 2a) and one-dimensional profiles of the surface relief were made in the radial direction starting from zone 1 to zone. Within each “window” about 12 one-dimensional profiles were analyzed to ensure representativeness of the data along the defect-induced relief with vertical resolution of 0.1 nm and horizontal resolution of \( \sim 0.5 \) \( \mu \)m.

A minimum (critical) scale \( l_{sc} \) that corresponds to beginning of multiscale long-range correlation in defect ensembles is determined by the computing of the Hurst exponent. The function \( K(r) \) is calculated for one-dimensional profiles of fracture surface relief according to the formula [5,6]:

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

where \( \Delta h(i) \) is the relief value at position \( i \), and \( \Delta h(i+r) \) is the relief value at position \( i+r \), and \( N \) is the number of segments.

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]

\[ K(r) = \frac{1}{r} \sum_{i=1}^{N} |\Delta h(i) - \Delta h(i+r)| \]
where \( K(r) \) is the averaged difference between the values of surface relief heights \( z(x+r) \) and \( z(x) \) in the window of size \( r \), and \( H \) is the Hurst exponent (surface roughness index).

Representation of the function \( K(r) \) in logarithmic coordinates allowed one to evaluate the lower boundary of the scaling range \( l_{sc} \), and the value of upper boundary considering one as the characteristic scale of the process zone \( L_{pz} \), i.e. the area of correlated behavior of multiscale defect structures (Fig.3).

Figure 2: Characteristic surface relief of a fatigue fracture zone: high cycle fatigue (a); gigacycle fatigue (b).

Figure 3: Characteristic one-dimensional profiles for zone 1 (a), plot \( \ln K(r) \) vs. \( \ln r \) for zone 1 (b).

The values of the Hurst exponent \( H \) and the scales \( L_{pz} \) and \( l_{sc} \) for different loading conditions are given in Tab. 1.

**CONCLUSION**

The comparative analysis of the scaling characteristics of samples loaded under conditions of high - and gigacycle fatigue shows a significant decrease in the range of spatial scales, where the Hurst exponent remains constant for dynamically loaded samples in the «fish-eye» (0.5-10.9 mkm) zone. This result confirms our assumption that mentioned characteristic scales \( L_{pz} \) and \( l_{sc} \) play an important role in the list of variables for the kinetic equation fatigue of
crack growth and can be used for the identification of phenomenological parameters (power index) providing self-similar law for the crack path.

<table>
<thead>
<tr>
<th>Sample number</th>
<th>Elongation (mm)</th>
<th>Striker velocity (m/s)</th>
<th>$\sigma$, MPa</th>
<th>$\Delta N$, cycles</th>
<th>Zone number</th>
<th>$L_{\alpha}$, mkm</th>
<th>$L_{pz}$, mkm</th>
<th>$H$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.89</td>
<td>28.4</td>
<td>130</td>
<td>7.33·10$^6$</td>
<td>2</td>
<td>1.4</td>
<td>20.6</td>
<td>0.57</td>
</tr>
<tr>
<td>2</td>
<td>1.77</td>
<td>40.3</td>
<td>120</td>
<td>7.82·10$^8$</td>
<td>2</td>
<td>0.5</td>
<td>10.9</td>
<td>0.63</td>
</tr>
<tr>
<td>3</td>
<td>1.27</td>
<td>32.1</td>
<td>120</td>
<td>5.72·10$^7$</td>
<td>2</td>
<td>1.0</td>
<td>18.2</td>
<td>0.60</td>
</tr>
<tr>
<td>4</td>
<td>2.21</td>
<td>40.3</td>
<td>105</td>
<td>5.83·10$^6$</td>
<td>2</td>
<td>1.0</td>
<td>14.2</td>
<td>0.46</td>
</tr>
</tbody>
</table>

Table 1: The values of the Hurst exponent $H$ and scales $L_{\alpha}$ and $L_{p}$ at various levels of fatigue longevity.

ACKNOWLEDGEMENT

This study was supported by the Russian Science Foundation, project № 14-19-01173.

REFERENCES