Visual Abstract

The Stress Intensity Factor of convex embedded polygonal cracks

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ABSTRACT. In the present work, a simple formula for the evaluation of the stress intensity factor (SIF) of convex embedded polygonal cracks has been proposed. This formula is structured as a correction factor of the Oore-Burns' equation and is based on accurate three-dimensional FE analysis. Furthermore, a precise formula for a regular polygonal crack has been given.

KEYWORDS. Weight function, Stress intensity factor, Three-dimensional crack.

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INTRODUCTION

The weight function technique was performed by Bueckner [1] and Rice [2, 3] to solve the stress intensity factor (SIF) of a crack as well as to find out the nominal stress over the geometric discontinuity. It is well known that the O-integral [4] is a good approximation to evaluate the SIF along the contour of three-dimensional planar cracks. For example, British Standard 7910 [5] suggests the use of the O-integral provided that the results are documented. Usually, in order to simplify SIF assessments for embedded flaws, the SIF is determined at the ends of both the minor and major axes of the elliptical idealization of the flaw [6, 7, 8]. The SIF of a semi elliptical crack can be accurately calculated by means of the general procedure of Shen and Glinka [9, 10]. They use four terms of approximation and evaluate the unknown parameters based on reference stress intensity factor expressions taken from the literature. In this way a general weight function can be formulated to overcome some problems [11, 12] in the Petrosky and Achenbach method [13] where an approximate displacement field from a known reference was calculated.

Also in Fitness-for-Service procedure crack shapes, idealisation becomes necessary when real cracks have been detected during an inspection. Typically, elliptical cracks, semi-elliptical cracks and through wall cracks or edge cracks with a rectilinear flank are considered [14]. For convex three-dimensional cracks few examples are presented in classical textbooks [15, 16, 17]. Furthermore, in order to obtain an acceptable approximation of the maximum SIF K_{Imax}, Murakami [18, 19] took into account many types of convex embedded cracks subjected to nominal tensile stress σ_n. The SIF was investigated numerically, and a final equation for SIF was given in the form: $K_{I,\max} = Y \sigma_n \sqrt{\pi \sqrt{A}}$ where Y is a coefficient that was evaluated as best fitting of the numerical results; and A is the area of the flaw.

In a previous paper [20] the authors investigated the failure of the O-integral in presence of high curvature for regular ($\partial\Omega$ of class C^2) cracks. In terms of regularity, a corner Q' means a singularity. This requires a new technique in order to correct the O-integral. In particular, in this paper we are interested in adjusting the O-integral for convex polygons. The analytical results will be compared with numerical ones obtained from an accurate three-dimensional FE analysis.

WEIGHT FUNCTION FOR A THREE-DIMENSIONAL CRACK: ANALYTICAL BACKGROUND

Oore and Burns proposed a general equation for the evaluation of the SIF of a two-dimensional crack inside a three-dimensional body subjected to a nominal tensile stress $\sigma_n(Q)$. The nominal stress $\sigma_n(Q)$ is evaluated without the presence of the crack. Q is the inner point of the crack. The crack can be considered as an open bounded simply connected subset Ω of the plane as reported in Fig. 1. We define:

$$f(Q)=\int_{\partial\Omega}\frac{ds}{\left|Q-P(s)\right|^2} \tag{1}$$

where $Q=Q(x,y)\in\Omega$, s is the arch-length parameter and point P(s) runs over the boundary $\partial\Omega$. In reference [4] Oore-Burns proposed an empirical formula for the evaluation of the mode I stress intensity factor at each point of the border crack of boundary $\partial\Omega$:

$$K_{I,OB}(Q') = \frac{\sqrt{2}}{\pi}\int_{\Omega}\frac{\sigma_n(Q)}{\sqrt{f(Q)}\,\left|Q-Q'\right|^2}d\Omega \quad , \qquad Q'\in\partial\Omega \tag{2}$$

Under reasonable hypotheses on the function $\sigma_n(Q)$, the integral (2) is convergent and the proof is based on the asymptotic behaviour of f(Q) [21].
In different papers, the authors analysed the properties of the Oore-Burns integral where the accuracy of the equation was tested in the particular case of an elliptical crack [22, 23].

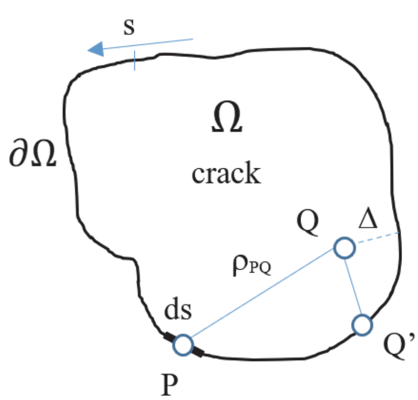

Figure 1: Inner crack.

CONVEX POLYGON

In a previous paper [20] we investigated the proprieties of the O-integral in the presence of high curvature for regular ($\partial\Omega$ of class C^2) cracks. Henceforth, we denote by $\xi=\xi(Q')$ the correction factor of Eqn. (2) at $Q'\in\partial\Omega$

$$K_I(Q') = \xi(Q')\;K_{I,OB}(Q') \tag{3}$$

In order to draw out the factor ξ, we need to take into account some hints:

A) a regular polygon with very large N sides, is "indistinguishable" from a disk, of which the SIF is given by $\frac{2\sigma_n}{\sqrt{\pi}}$ times the square root of the disk radius;
B) ξ is close to one away from the corners;
C) known FE results for cracks with a different shape: square, equilateral triangles and rectangular (aspect ratio 1/3).
D) the crack is subjected to a uniform tensile stress σ_n.

The key to construct ξ(Q') is a smart use of the hyperbolic tangent function x→tanh(x). At an early stage, the factor ξ will contain some unknown parameters that will be carefully calibrated on the basis of requirements A, B and C. Let us assume p is the perimeter of the convex polygon Ω and c is the perimeter of the smallest disk containing Ω. Near a fixed edge P with opening angle α (see Fig. 2, where $x_0 = |Q' - P|, \omega = \pi - \alpha$), ξ(Q') can be given as follows:

$$\xi(\mathrm{Q'}) = \left(1 + \tau\sqrt{\frac{c}{p} - 1}\right)\tanh\left[\mu\left(\frac{c}{p}\right)^{\beta}\left(\frac{\pi}{2\omega}\right)^{\beta}\left(\frac{x_0}{p}\right)^{\gamma}\right] \tag{4}$$

with τ, μ, β and γ chosen in order to satisfy A), B) and C) conditions.
On the basis of accurate FE analysis on a three-dimensional model as proposed in reference [20], the best agreement is given by the choice $\tau = \frac{1}{10}$, μ=6.95, β=0.8 and γ=0.4. This means that near the corner, the coefficient ξ(Q') will be close to the value:

$$\xi(\mathrm{Q'}) = \left(1 + \frac{1}{10}\sqrt{\frac{c}{p} - 1}\right)\tanh\left[6.95\left(\frac{c}{p}\right)^{0.8}\left(\frac{\pi}{2\omega}\right)^{0.8}\left(\frac{x_0}{p}\right)^{0.4}\right] \tag{5}$$

Now, it is possible to extend Eqn. (5) to entire contour ∂Ω in a natural way, by taking into account all distances (on the geometry of ∂Ω) of Q' from each corner of the polygon:

$$\xi(\mathrm{Q'}) = \begin{aligned} &\left(1 + \frac{1}{10}\sqrt{\frac{c}{p} - 1}\right)\prod_{1}^{N}\tanh\left[6.95\left(\frac{c}{p}\right)^{0.8}\left(\frac{\pi}{2\omega_k}\right)^{0.8}\left(\frac{|Q' - P_k|}{p}\right)^{0.4}\right] \\ &= \left(1 + \frac{1}{10}\sqrt{\frac{c}{p} - 1}\right)\prod_{1}^{N}\tanh\left[6.95\left(\frac{c\,\pi\sqrt{|Q' - P_k|}}{2\omega_k\, p^{3/2}}\right)^{0.8}\right] \end{aligned} \tag{6}$$

where P_k, k=1, 2, ..., N are the corners with opening angles α_k, $\omega_k = \pi - \alpha_k$ and $|Q' - P|$ is the distance between Q' and P_k on the boundary ∂Ω.

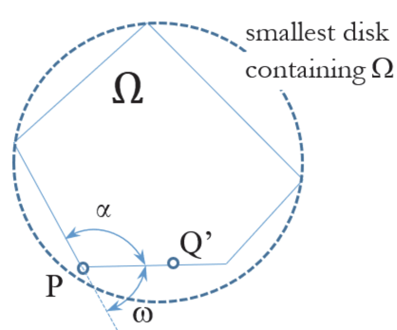

Figure 2: Polygonal crack.

For example, we explain Eqn. (6) on an equivalent triangle with side L, where $Q' = \left(\frac{L}{2\sqrt{3}}, y_0 \right)$ and $0 \le \frac{y_0}{L} \le 0.5$ as shown in Fig. (3):

$$\xi(Q') = 1.046 \tanh\left[4.14 \left(\frac{1}{2} - \frac{y_0}{L} \right)^{0.4} \right] \cdot \tanh\left[4.14 \left(\frac{3}{2} - \frac{y_0}{L} \right)^{0.4} \right] \cdot \tanh\left[4.14 \left(\frac{1}{2} + \frac{y_0}{L} \right)^{0.4} \right] \tag{7}$$

Clearly, in the r.h.s. of the Eqn. (7), the only significant contribution is given by

$$\xi(Q') = 1.046 \tanh\left[4.14 \left(\frac{1}{2} - \frac{y_0}{L} \right)^{0.4} \right] \tag{8}$$

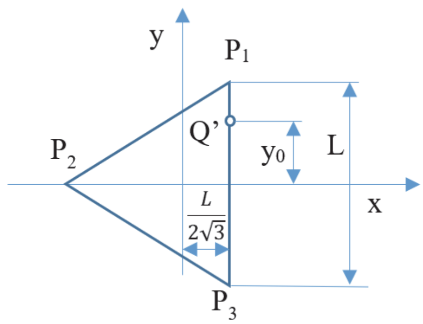

Figure 3: Equilateral triangular crack.

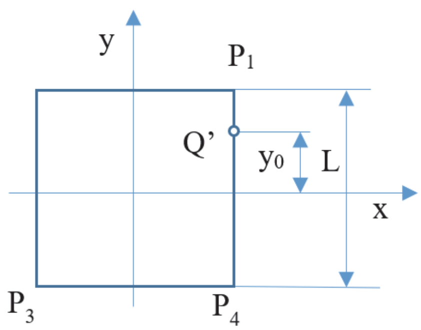

Figure 4: Square crack.

As a second example, Fig. 4 shows the case of a square crack of side L, where $Q' = \left(\frac{1}{2} L , y_0 \right)$ and $0 \le \frac{y_0}{L} \le 0.5$. Then

$$\xi(Q') = \begin{array}{c} 1.033 \cdot \tanh\left[4.34 \left(\frac{1}{2} - \frac{y_0}{L} \right)^{0.4} \right] \cdot \tanh\left[4.34 \left(\frac{3}{2} - \frac{y_0}{L} \right)^{0.4} \right] \cdot \\ \tanh\left[4.34 \left(\frac{3}{2} + \frac{y_0}{L} \right)^{0.4} \right] \cdot \tanh\left[4.34 \left(\frac{1}{2} + \frac{y_0}{L} \right)^{0.4} \right] \end{array} \tag{9}$$

In the r.h.s. of the Eqn. (8), the only significant contribution is given by

$$\xi(Q') = 1.033 \tanh\left[4.34 \left(\frac{1}{2} - \frac{y_0}{L} \right)^{0.4} \right] \tag{10}$$

Eqns. (8) and (10) were recently announced in a very similar preliminary form in paper [20]. We remark that Eqn. (6) can be reasonably extended to every convex set with corners.
In the case of a square-like flaw, the Oore-Burns integral can be analytically expressed in simplified form in the middle of the side and in the middle of the rounded corner [24]. The Oore-Burns integral will be approximated by means of Riemann sums plus a suitable asymptotic correction in terms of mesh size [25, 26].

Asymptotic behaviour of the SIF on regular polygons

Let Ω be a regular polygon with N sides, inscribed in the disk of radius *a*. This means that the length of the side is $L = 2 \cdot a \cdot \sin\left(\frac{\pi}{N}\right)$, $\omega_k = 2\pi/N$. Then (see Fig. 5) from Eqn. (6) after some simple steps:

$$\xi(Q') \sim \tanh\left[1.74\left((1-\lambda)\cdot N\right)^{0.4}\right] \tag{11}$$

where $Q' = \left(cos\left(\frac{\pi}{N}\right)a,\ y_0\right)$, $y_0 = \lambda \cdot \frac{L}{2}$, $0 \le \lambda \le 1$.

Taking into account that on the unitary disk, the O-Integral takes the value $\frac{2\sigma_n}{\sqrt{\pi}}$, it follows the asymptotic behavior of the SIF, for N→∞:

$$K_I \sim \frac{2\,\sigma_n\sqrt{a}}{\sqrt{\pi}}\tanh\left[1.74\left((1-\lambda)\cdot N\right)^{0.4}\right] \tag{12}$$

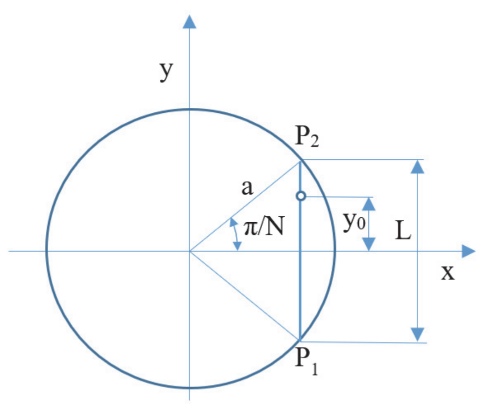

Figure 5: Polygonal crack.

Fig. 6 shows the graph of the SIF in dimensionless form for N equal to 20. In the middle of the side the SIF tends to have a constant value near to the one of a circular crack. At the corner the value of the SIF is null and in the proximity of the corner the trend shows a cup as calculated in reference [20].
In order to check Eq. (12), an accurate numerical FE analysis has been proposed in a case of a regular hexagonal crack. Figs. 7 and 8 show a three-dimensional model and the FE model, respectively. The mesh is refined only near the point of interest where the SIF is evaluated as proposed in previous works [22–25]. The dimensions of smaller elements at the tip of the crack were in the order of 10^{-5} mm. Finally, Fig. 9 proposes the comparison of the SIF in dimensionless form evaluated along the border in the range [0, L/2]. Eq. (12) appears precise and modifies the value of the SIF only near the corner.
The value of $K_{I,OB}$ was evaluated by means of the procedure valid for a star domain proposed in reference [26] where a mesh with a step of 0.0157 radiant in the tangential direction and 80 terms in the Fourier series have been considered.

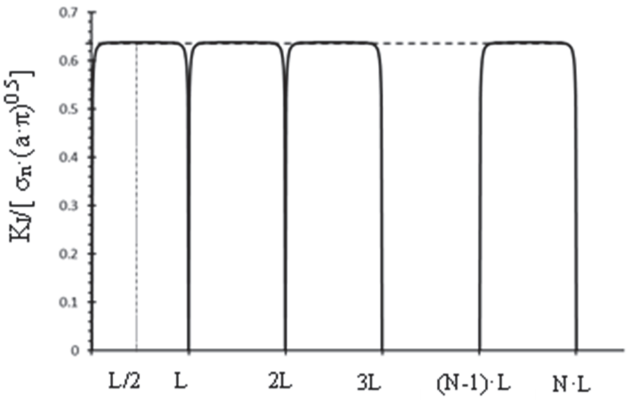

Figure 6: Stress intensity factor in dimensionless form for a regular polygonal crack with N=20 (L/a=0.313) under uniform tensile load.

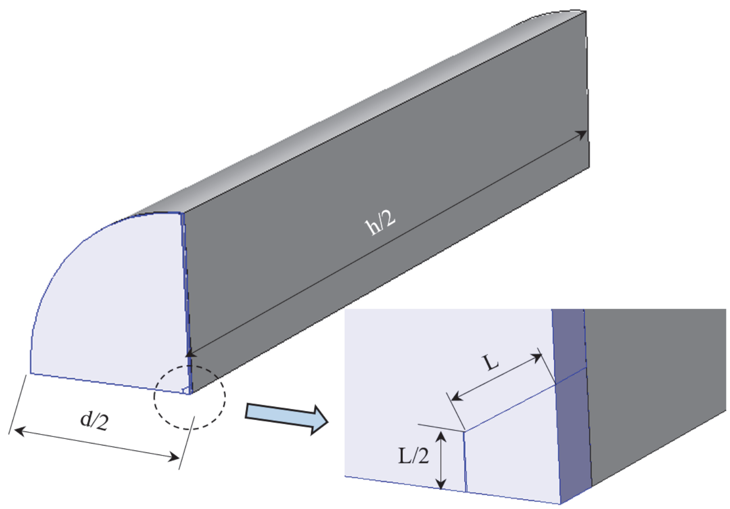

Figure 7: Three-dimensional model for a hexagonal crack (L=1 mm, h/L= 200, d/L=20).

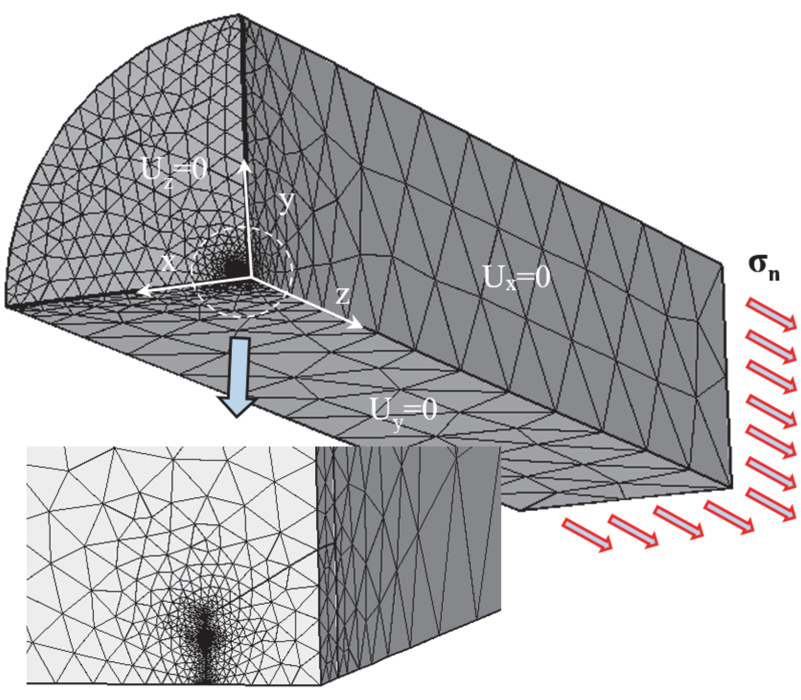

Figure 8: Mesh and boundary conditions for a hexagonal crack of Fig. 7.

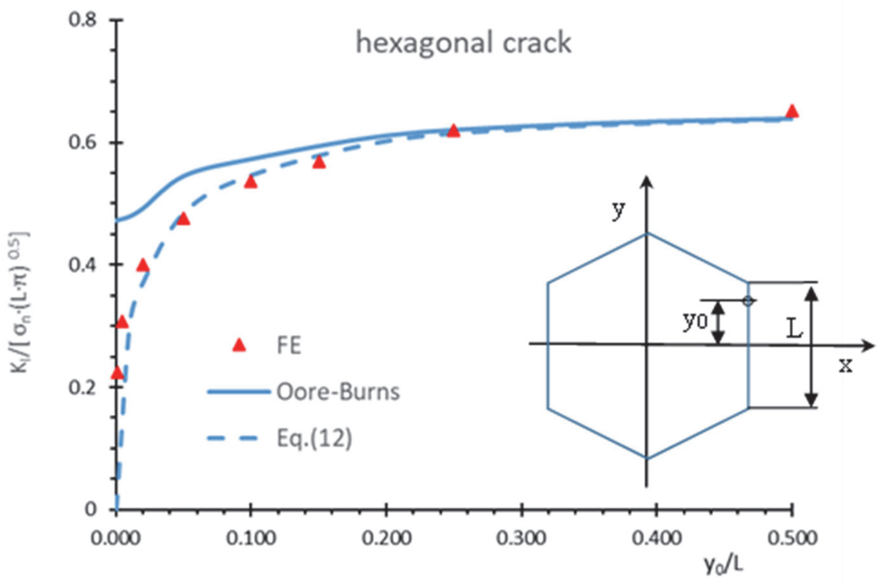

Figure 9: Stress intensity factor in dimensionless form for a hexagonal crack under uniform tensile load.

Conclusions

In this paper an accurate correction factor that multiplies the Oore-Burns stress intensity factor (SIF) is given for embedded polygonal cracks. This correction factor, ranging from zero up to one, essentially depends on the distance from the nearest corner, and it is of fundamental importance near the corner where the value of the Oore-Burns SIF does not reach a null value. Far from the corner, the correction factor quickly increases up to one. Furthermore, an accurate formula for regular polygonal cracks is given. A comparison with the FE results for hexagonal cracks shows satisfactory results.

Nomenclature

α	opening angle
a	crack size, disk radius
Y	shape factor
Ω	crack shape
$\partial\Omega$	crack border
Q	point of Ω
Q'	point of crack border
K_I	mode I stress intensity factor
$K_{I,OB}$	mode I stress intensity factor calculated with the Oore-Burns' equation
N	number of corners
ω	complementary angle
s	arc length
σ_n	nominal tensile stress in the x, y Cartesian coordinate system
p	perimeter of Ω
c	perimeter of the smallest disk containing Ω
ξ	correction factor
x, y	actual Cartesian coordinate system

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