



## Limit load based evaluation of plastic $\eta$ factor for C(T) specimen with a mismatched weld

R. Nikhil, S.A. Krishnan, G. Sasikala, A. Moitra

Materials development and Technology Division, Indira Gandhi Centre for Atomic Research,  
Kalpakkam 603102, Tamil Nadu, India

*rnikhil@igcar.gov.in, sakrish@igcar.gov.in, gsasi@igcar.gov.in, moitra@igcar.gov.in*



**ABSTRACT.** Plastic  $\eta$  factor is adopted to account for crack tip plasticity while evaluating the fracture toughness of the materials as per ASTM E1820. It is valid only for homogeneous materials. The plastic  $\eta$  factor for Compact Type (C(T)) geometry with type 316LN stainless steel weld has been evaluated based on elastic-plastic FE analysis. The incremental elastic-plastic material model with various values for strength mismatch ratio ( $M$ ) i.e. ratio of yield strength of weld metal to that of base metal, from 1.2 to 2.2 have been considered. The weld width ( $b$ ) parallel to the crack plane is varied from 4 mm to 16 mm. The  $\eta$  values thus obtained are analyzed and the inferences are discussed.

**KEYWORDS.** Limit load; Plastic  $\eta$ ; Elastic-plastic; Strength mismatch level.

**Citation:** Nikhil, R., Krishnan, S.A., Sasikala, G., Moitra, A., Limit load based evaluation of plastic  $\eta$  factor for C(T) specimen with a mismatched weld, *Frattura ed Integrità Strutturale*, 48 (2019) 523-529.

**Received:** 28.11.2018

**Accepted:** 28.02.2019

**Published:** 01.04.2019

**Copyright:** © 2019 This is an open access article under the terms of the CC-BY 4.0, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

## INTRODUCTION

Weld joints are more likely to contain flaws or defects. The fracture toughness of weldment under the influence of base material strength is an essential input for integrity assessment of welded structures.  $J$ -integral as a measure of fracture toughness is generally determined according to ASTM E1820. It is a fracture characterizing parameter based on non-linear elastic material model applied to elastic-plastic materials. The  $J$ -integral is divided into elastic (linear),  $J_e$  and plastic (non-linear),  $J_p$  components. The elastic component of  $J$  ( $J_e = \frac{K^2}{E}$ ) is same as Griffith energy release rate and

the plastic part is given ( $J_p = \frac{\eta A_p}{Bb}$ ), where  $\eta$  is a geometry normalizing parameter to account crack tip plasticity i.e. spread of plastic zone around the crack tip. It is influenced by geometry of the test specimen. For compact tension (CT) specimens, ASTM has adopted an expression (eqn. 1) as obtained by Landes and Clarke [1].

$$\eta = 2 + 0.522 \left( \frac{b}{W} \right) \quad (1)$$



This expression is valid for specimens made of homogenous material. Wang and co-authors [2] proposed CTOD equations expressed in terms of weld height, mismatch level and strain hardening rate for specimens made of non-homogeneous and strain hardening materials. Smith [3], Panontin and co-authors [4], Cassanelli and co-authors [5], Kim and co-authors [6], Davies and co-authors [7], have proposed analytical and numerical solution to  $\eta$  expression for even match ( $M=1$ ) C(T) specimens but varying strain hardening exponent. Xuan and co-authors [8] and Marie & Nedelev [9] have performed FE based analysis for various mismatch factors from 0.25 to 2 and 2.3 respectively. A literature review of  $\eta$  solution for C(T)specimen in tabular form is provided by H. Zhou and co-authors [10]. They have analyzed the influence of mismatch factor, weld height, material hardening exponent and  $a/W$  ratio effect on  $\eta$  solution.

ASTM Type 316LN stainless steel is a major structural material for fast breeder reactor being commissioned at Kalpakkam, India. The  $M$  value for 316LN weld is found to vary across the weld thickness [11] and found to be as high as 2.2. In the present study, plane strain FE analysis have been carried out to assess the  $\eta$  factor for C(T) geometry with weld width to specimen width ratio,  $b/W$  varying from 0.08 to 0.32 and crack depth to width ratio,  $a/W$  varying from 0.45 to 0.7 for strength mismatch  $M = 1.2, 1.4, 1.6, 1.8, 2$  and 2.2.

## THEORETICAL BACKGROUND

**F**or homogeneous material, as per ASTM experimental load, load line displacement (crack opening displacement) and crack length data are obtained to evaluate  $J$ -integral values for a growing crack. A typical plot is shown in Fig. 1.

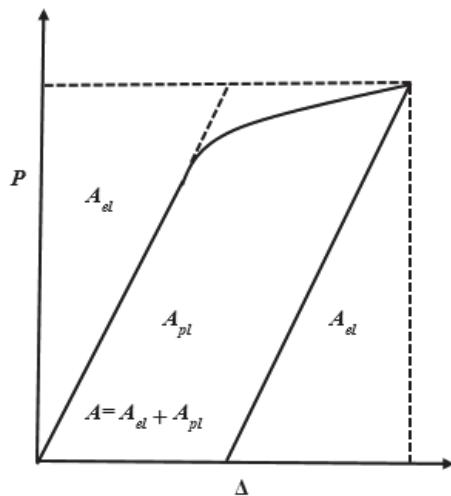


Figure 1: Area under the load-displacement plot.

$J$ -integral consists of an elastic component and a plastic component.

$$J = J_e + J_p \quad (2)$$

where

$$J_e = \frac{K^2}{E'} \text{ such that } E' = E \text{ for plane stress}$$

$$E' = \frac{E}{(1-\nu^2)} \text{ for plane strain} \quad (3)$$

$$J_p = \frac{\eta A_p}{Bb} \quad (4)$$

As per Ernst et al. [12,13], if limit load ( $P_L$ ) can be expressed in terms of independent functions of crack length ( $a$ ) and load-line displacement ( $\Delta_{pl}$ ) then  $\eta$  can be calculated based on  $P_L$ .

$$P_L = F(a)G(\Delta_{pl}) \quad (5)$$

Assuming the material behavior to be ideal plastic, Chattopadhyay et al. [14] proposed  $\eta$  as

$$\eta = -\frac{1}{P_L} \frac{\partial P_L}{\partial a} \quad (6)$$

knowing  $P_L$ , the eq. (6) issued for heterogeneous C(T) specimens.  $P_L$  could be evaluated by analytical solutions available in open literature [15] or Twice elastic slope (TES) / Twice elastic deflection (TED) or FE based yield contour (FYC) plot across the ligament. In present study  $P_L$  has been obtained (i) based on TES method from FEM simulated load-displacement plots and (ii) FE-yield contour plot across the ligament of C(T) specimen.

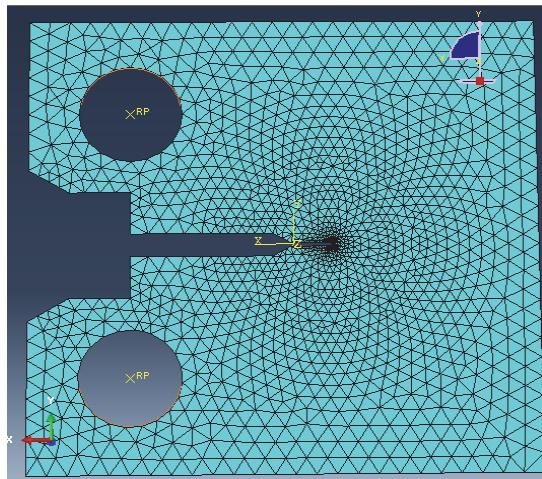


Figure 2: Meshed CT geometry with constraints.

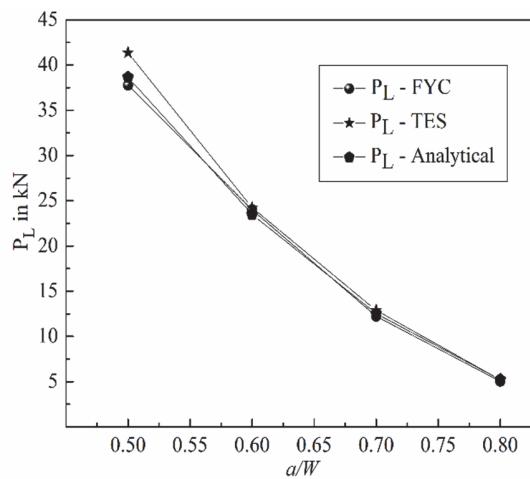


Figure 3: Limit load vs  $a/W$  using various approaches.

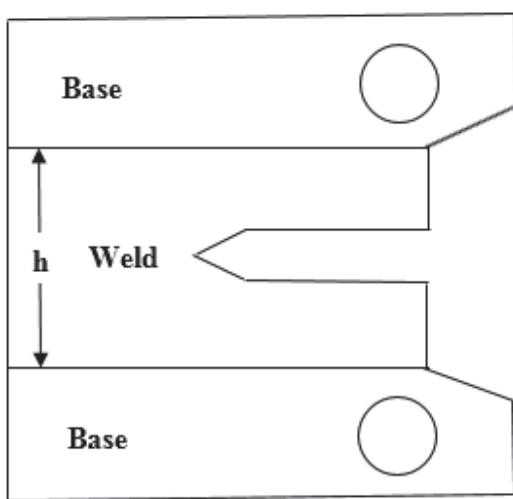


Figure 4: Schematic of weld C(T) specimen.

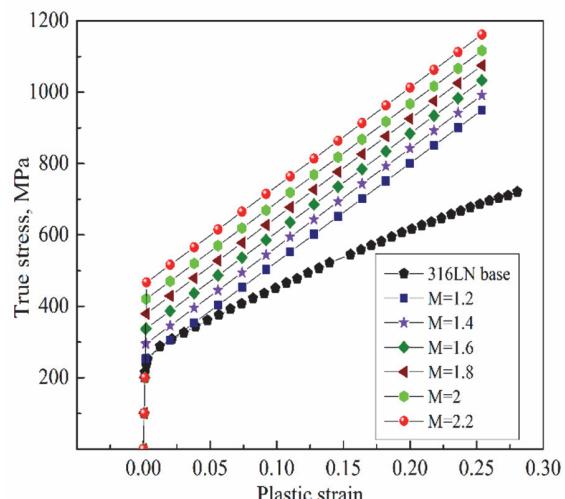
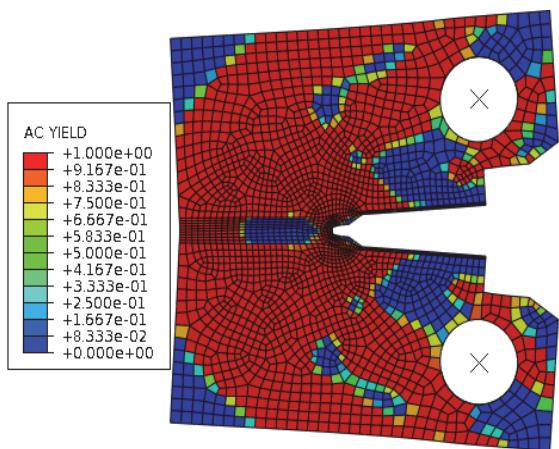
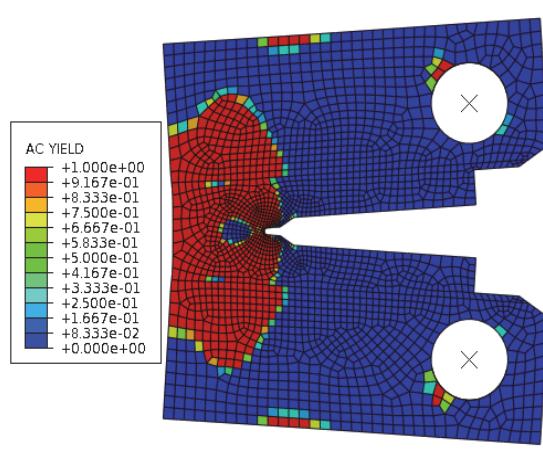
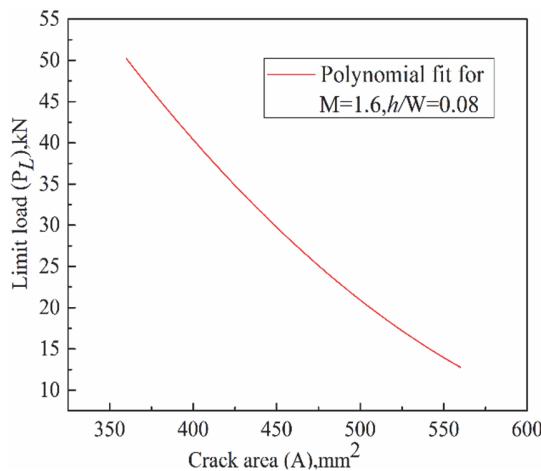
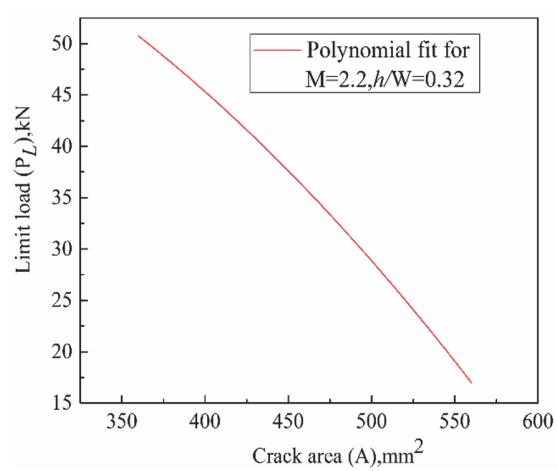


Figure 5: Bilinear stress strain plot.

Figure 6(a): AC yield plot for  $a/W=0.5$ ,  $b/W=0.08$  and  $M=1.6$ Figure 6(b): AC yield plot for  $a/W=0.7$ ,  $b/W=0.32$  and  $M=2.2$ Figure 7(a):  $P_L$  vs  $A$  for  $b/W=0.08$  and  $M=1.6$ Figure 7(b):  $P_L$  vs  $A$  for  $b/W=0.32$  and  $M=2.2$ 

## FEM ANALYSIS

**2** D FE analysis for standard C(T) geometry with base and base-weld metal configuration consisting of  $b/W$  ratio (0.08, 0.16, 0.24 and 0.32),  $a/W$  (0.45, 0.5, 0.55, 0.6, 0.65 and 0.7) and  $M$  (1.2, 1.4, 1.6, 1.8, 2 and 2.2) has been carried out using ABAQUS. Loading pins of the specimen are modeled as rigid bodies and loaded by applying displacement while all other motions of the pins are restrained. Surface-to-surface contact with a finite-sliding formulation is defined between the pins and the specimen hole. A typical C(T) geometry mesh model with constraints highlighted is shown in Fig. 2. As per ASTM E1820, high stress triaxiality at crack tip is ensured in C(T) specimens by side grooving. Hence in the present study the analysis is restricted plane strain (CPE4) condition. Elastic-perfect plastic simulations have been carried out to evaluate limit load for homogeneous C(T) specimens with  $a/W$  ratios (0.45, 0.5, 0.55, 0.6, 0.65 and 0.7). Flow stress of 410 MPa has been input for analysis. The limit load obtained using analytical formula, Twice Elastic Slope (TES) and FE based Yield Contour (FYC) approaches are compared. FYC is obtained using AC Yield parameter in ABAQUS which provides the extent of yielding of various elements. The  $P_L$  values are shown in Fig. 3. It is observed that the  $P_L$  values based FYC are in good agreement with those obtained from TES and analytical solutions.

Therefore, the FEM procedure adopted to evaluate the limit load could be extended to the heterogeneous C(T) specimens. Towards this, Elastic-plastic simulations have been carried out for C(T) specimens with weld width as shown in Fig. 4. The yield stress, UTS and % elongation obtained from all weld tensile test is 462 MPa, 658 MPa and 28% elongation respectively. Based on these values a bi-linear true stress-plastic strain data generated considering identical hardening behaviour for all  $M$  values as shown in Fig. 5 is used as material model input. A typical FYC corresponding to the limit load obtained for weld specimen with  $b/W = 0.08$ ,  $a/W = 0.5$ ,  $M = 1.6$  is shown in Fig. 6(a) and for  $b/W = 0.32$ ,  $a/W = 0.7$ ,  $M = 2.2$  is shown in Fig. 6(b). For a given load line displacement, the spread of yield contour is attributed to  $M$  and  $b$ , in case of specimen with



$M=1.6$  and  $b=4$  mm, the yield contour crosses the interface boundary and spreads to base material to a larger area compared to specimen with  $M=2.2$  and  $b=16$  mm.

## RESULTS AND DISCUSSION

Typical  $P_L$  vs. A plot for  $M=1.6$ ,  $b/W=0.08$  and  $M=2.2$ ,  $b/W=0.32$  is shown in Figs. 7(a)and 7(b). These values are fitted to a second order polynomial. As per the eqn. 6,  $\eta$  values are calculated for all configurations of C(T)specimens.

For the homogeneous C(T)specimens, the calculated  $\eta$  values are in close agreement of  $\pm 4\%$  with ASTM proposed  $\eta$  values as given eq. (1). Similar observation has been reported by Zhou and co-authors [10]. The estimated  $\eta$  values for various configurations of weld C(T)specimen are given in Table 1.

$\eta$ for $M=1.2$					$\eta$ for $M=1.8$					$\eta$ for $M=2$					$\eta$ for $M=2.2$									
$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32
$b/W \downarrow$					$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$			
0.3	2.096	1.910	1.809	2.338	0.3	2.081	1.795	1.597	1.767	0.3	1.866	1.970	1.767	1.550	0.3	1.868	1.751	1.640	1.536	0.3	2.082	1.834	1.815	1.838
0.35	2.185	2.106	1.975	2.202	0.35	2.130	1.892	1.746	1.787	0.35	2.041	1.942	1.823	1.649	0.35	2.034	1.832	1.713	1.633	0.35	2.140	2.003	1.886	1.798
0.4	2.231	2.185	2.121	2.116	0.4	2.153	1.906	1.795	1.767	0.4	2.101	1.929	1.801	1.695	0.4	2.088	1.898	1.752	1.674	0.4	2.168	2.056	1.864	1.854
0.45	2.262	2.239	2.173	2.040	0.45	2.170	2.001	1.952	1.758	0.45	2.146	1.875	1.824	1.756	0.45	2.128	1.980	1.794	1.705	0.45	2.188	2.098	1.904	1.903
0.5	2.302	2.303	2.238	2.008	0.5	2.202	2.036	1.947	1.778	0.5	2.199	1.856	1.849	1.799	0.5	2.181	1.956	1.845	1.761	0.5	2.251	2.215	2.046	1.928
0.55	2.259	2.273	2.225	1.993	0.55	2.169	2.019	1.985	1.815	0.55	2.189	1.880	1.851	1.819	0.55	2.122	2.049	1.909	1.778	0.55	2.213	2.197	2.038	1.916
$\eta_{mean}$	2.223	2.169	2.090	2.116	$\eta_{mean}$	2.151	1.941	1.837	1.778	$\eta_{mean}$	2.090	1.908	1.819	1.711	$\eta_{mean}$	2.164	2.049	1.885	1.862	$\eta_{mean}$	2.079	1.903	1.765	1.681
$\eta$ for $M=1.4$					$\eta$ for $M=2$					$\eta$ for $M=2.2$					$\eta$ for $M=1.6$					$\eta$ for $M=2.2$				
$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32
$b/W \downarrow$					$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$			
0.3	2.088	1.869	1.735	2.121	0.3	1.866	1.970	1.767	1.550	0.3	2.082	1.834	1.815	1.838	0.3	1.868	1.751	1.640	1.536	0.3	2.082	1.834	1.815	1.838
0.35	2.156	2.045	1.829	2.029	0.35	2.041	1.942	1.823	1.649	0.35	2.140	2.003	1.886	1.798	0.35	2.034	1.832	1.713	1.633	0.35	2.140	2.003	1.886	1.798
0.4	2.190	2.108	1.911	1.991	0.4	2.101	1.929	1.801	1.695	0.4	2.168	2.056	1.864	1.854	0.4	2.088	1.898	1.752	1.674	0.4	2.168	2.056	1.864	1.854
0.45	2.214	2.154	2.003	1.938	0.45	2.146	1.875	1.824	1.756	0.45	2.188	2.098	1.904	1.903	0.45	2.128	1.980	1.794	1.705	0.45	2.188	2.098	1.904	1.903
0.5	2.251	2.215	2.046	1.928	0.5	2.199	1.856	1.849	1.799	0.5	2.222	2.157	1.918	1.881	0.5	2.181	1.956	1.845	1.761	0.5	2.222	2.157	1.918	1.881
0.55	2.213	2.197	2.038	1.916	0.55	2.189	1.880	1.851	1.819	0.55	2.188	2.148	1.924	1.897	0.55	2.172	2.002	1.846	1.777	0.55	2.188	2.148	1.924	1.897
$\eta_{mean}$	2.185	2.098	1.927	1.987	$\eta_{mean}$	2.090	1.908	1.819	1.711	$\eta_{mean}$	2.164	2.049	1.885	1.862	$\eta_{mean}$	2.079	1.903	1.765	1.681	$\eta_{mean}$	2.164	2.049	1.885	1.862
$\eta$ for $M=1.6$					$\eta$ for $M=2.2$					$\eta$ for $M=1.6$					$\eta$ for $M=2.2$					$\eta$ for $M=1.6$				
$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32	$b/W \rightarrow$	0.08	0.16	0.24	0.32
$b/W \downarrow$					$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$				$b/W \downarrow$			
0.3	2.082	1.834	1.815	1.838	0.3	1.868	1.751	1.640	1.536	0.3	2.082	1.834	1.815	1.838	0.3	1.868	1.751	1.640	1.536	0.3	2.082	1.834	1.815	1.838
0.35	2.140	2.003	1.886	1.798	0.35	2.034	1.832	1.713	1.633	0.35	2.140	2.003	1.886	1.798	0.35	2.034	1.832	1.713	1.633	0.35	2.140	2.003	1.886	1.798
0.4	2.168	2.056	1.864	1.854	0.4	2.088	1.898	1.752	1.674	0.4	2.168	2.056	1.864	1.854	0.4	2.088	1.898	1.752	1.674	0.4	2.168	2.056	1.864	1.854
0.45	2.188	2.098	1.904	1.903	0.45	2.128	1.980	1.794	1.705	0.45	2.188	2.098	1.904	1.903	0.45	2.128	1.980	1.794	1.705	0.45	2.188	2.098	1.904	1.903
0.5	2.222	2.157	1.918	1.881	0.5	2.181	1.956	1.845	1.761	0.5	2.222	2.157	1.918	1.881	0.5	2.181	1.956	1.845	1.761	0.5	2.222	2.157	1.918	1.881
0.55	2.188	2.148	1.924	1.897	0.55	2.172	2.002	1.846	1.777	0.55	2.188	2.148	1.924	1.897	0.55	2.172	2.002	1.846	1.777	0.55	2.188	2.148	1.924	1.897
$\eta_{mean}$	2.164	2.049	1.885	1.862	$\eta_{mean}$	2.079	1.903	1.765	1.681	$\eta_{mean}$	2.164	2.049	1.885	1.862	$\eta_{mean}$	2.079	1.903	1.765	1.681	$\eta_{mean}$	2.164	2.049	1.885	1.862

Table 1:  $\eta$  values calculated for heterogeneous specimens using FYC approach.



For a given strength mismatch level, the  $\eta$  values have been found to decrease as the weld width is increased from 4 mm to 16 mm. This result is in agreement with literature [9,10]. Further the  $\eta$  values have been found to decrease with strength mismatch except between  $M=2$  &  $2.2$  where they are quite insensitive to mismatch. The similar trend has been reported in literature [10]. The  $\eta$  values vary with configurations, however the mean  $\eta$  value follow a trend line for  $M=1.6, 1.8, 2.0$  and  $2.2$ . The mean  $\eta$  value for  $M=1.2$  is nearly constant for various  $b/W$  ratios, for  $M=1.4$  it varies between 2.185 to 1.927. It is also observed that  $\eta$  decreases monotonically with increasing  $M$ , except for intermediate weld width i.e.  $b/W = 0.16$  and  $0.24$  as shown in Fig. 8. For C(T) configurations with  $M=1.6$  to  $2.2$ , there is a similarity and decreasing trend as the  $b/W$  ratio increases. The maximum mean  $\eta$  value is 2.16 and the minimum of 1.68.

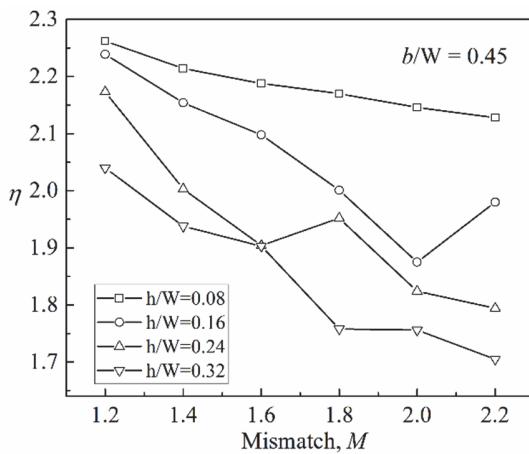


Figure 8:  $\eta$  vs  $M$  for  $b/W=0.45$ .

## CONCLUSION

The following important conclusions are drawn based on present study.

1. A validated finite element analysis yield contour (FYC) approach based on elastic-plastic material model is used to estimate limit load.
2. The plastic  $\eta$  factors for various configurations of C(T) specimen with weldment parallel to crack plane has been proposed to evaluate plastic  $J$ -integral.
3. For smaller strength mismatch ratio,  $M \leq 1.4$  the variation of mean  $\eta$  value is 2.2 to 1.93 hence evaluating configuration specific plastic  $\eta$  factor may not influence severe on  $J$ -estimation.
4. For larger values of  $M > 1.4$  the plastic  $\eta$  factor vary from 2.16 to 1.68, hence the values are C(T) configuration specific.

## REFERENCES

- [1] Clarke, G.A. and Landes, J.D. (1979). Evaluation of the J Integral for the Compact Specimen, Journal of Testing and Evaluation, JTEVA, 7(5), pp. 264–269. DOI:10.1520/JTE10222J.
- [2] Wang, Y.Y., Reemsnyder, H.S. and Kirk, M.T. (1997). Inference equations for fracture toughness testing: numerical analysis and experimental verification, ASTM Special Technical Publication (1321), pp. 469–484. DOI: 10.1520/STP12325S .
- [3] Smith, E. (1992). The use of eta factors to describe the J integral: the ASTM 1152 standard for the compact tension specimen, Engineering Fracture Mechanics, 41(2), pp. 241-246. DOI: 10.1016/0013-7944(92)90184-G.
- [4] Panontin, T.L., Makino, A. and Williams, J.F. (2000). Crack tip opening displacement estimation formulae for C(T) specimens, Engineering Fracture Mechanics, 67, pp. 293-301. DOI: 10.1016/S0013-7944(00)00048-5.
- [5] Cassanelli, A.N., Cocco, R. and de Vedia, L.A. (2003). Separability property and  $\eta_{pl}$  factor in ASTM A387-Gr22 steel plate, Engineering Fracture Mechanics, 70(9), pp. 1131-1142. DOI: 10.1016/S0013-7944(02)00095-4.
- [6] Kim, Y.J., Kim, J.S. and Cho, S.M. (2004). 3-Dconstraint effects on J testing and crack tip constraint in M(T), SE(B), SE(T) and C(T) specimens: numerical study, Engineering Fracture Mechanics, 71(9–10), pp. 1203–1218. DOI: 10.1016/S0013-7944(03)00211-X.



- [7] Davies, C., Kourpetis, M., O'Dowd, N. and Nikbin, K.M. (2007). Experimental evaluation of the J or C\* parameter for a range of cracked geometries, *Fatigue & Fracture Mechanics*, 35, pp. 321–340. DOI: 10.1520/JAI13221.
- [8] Xuan, F.Z., Tu, S.T. and Wang, Z.D. (2005). A modification of ASTM E 1457 C\* estimation equation for compact tension specimen with a mismatched cross-weld, *Engineering Fracture Mechanics*, 72(17), pp. 2602–2614. DOI: 10.1016/j.engfracmech.2005.05.002.
- [9] Marie, S. and Nedelec, M. (2011). Mismatch effect on C(T)specimen mechanical effect and consequences on the weld toughness characterization, In ASME conference proceedings, 44533, pp. 449–458. DOI: 10.1115/PVP2011-57168.
- [10] Zhou, H., Biglari, F., Davies, C.M., Mehmanparast, A. and Nikbin, K.M. (2014). Evaluation of fracture mechanics parameters for a range of weldment geometries with different mismatch ratios, *Engineering Fracture Mechanics*, 124, pp. 30–51. DOI: 10.1016/j.engfracmech.2014.03.006.
- [11] Hongo, H., Yamazaki, M., Watanabe, T., Tabuchi, M. and Albert, S.K. (2014). Evaluation of local fluctuation of creep properties in weld joint of 18Cr-12Ni-Mo-medium N - low C steel, *Pressure Vessels and Piping: Manufacturing and Performance*, Narosa Pub., pp. 307-316.
- [12] Ernst, H.A. and Paris, P.C. (1980). Techniques of analysis of load–displacement records by J-integral methods, US Nuclear Regulatory Commission, NUREG/CR 1222.
- [13] Paris, P.C., Ernst, H.A. and Turner, C.E. (1980). In: *Fracture mechanics, twelfth conference*, ASTM STP 700, Philadelphia, American Society for Testing and Materials, pp. 338–351. DOI: 10.1520/STP700-EB.
- [14] Chattopadhyay, J., Dutta, B.K. and Kushwaha, H.S. (2004). New ‘ $\eta_{pl}$ ’ and ‘T’ functions to evaluate J-R curves from cracked pipes and elbows: Part I—theoretical derivation, *Engineering Fracture Mechanics* 71, pp. 2635–2660. DOI: 10.1016/j.engfracmech.2004.01.011.
- [15] Kumar, V., German, M.D. and Shih, C.F. (1981). An engineering approach for elastic–plastic fracture analysis, EPRI-NP-1931, Project 1287-1, Topical Report, Electric Power Research Institute, Palo Alto, CA.