Mode II brittle fracture: recent developments

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ABSTRACT. Fracture behaviour of V-notched specimens is assessed using two energy based criteria namely the averaged strain energy density (SED) and Finite Fracture Mechanics (FFM). Two different formulations of FFM criterion are considered for fracture analysis. A new formulation for calculation of the control radius R_c under pure Mode II loading is presented and used for prediction of fracture behaviour. The critical Notch Stress Intensity Factor (NSIF) at failure under Mode II loading condition can be expressed as a function of notch opening angle. Different formulations of NSIFs are derived using the three criteria and the results are compared in the case of sharp V-notched brittle components under in-plane shear loading, in order to investigate the ability of each method for the fracture assessment. For this purpose, a bulk of experimental data taken from the literature is employed for the comparison among the mentioned criteria.

KEYWORDS. Fracture strength; Sharp V-notch; mode II fracture; Notch stress intensity factor; Strain energy density.

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INTRODUCTION

Numerous failure criteria have been proposed by scholars dealing with the brittle fracture of V-notches [1]. According to the intensified linear elastic stress at the notch tip, introducing a stress field parameter is useful. Considering the Linear Elastic Notch Mechanics (LEFM) concept, NSIFs can be successfully employed for the fracture assessment of brittle materials when weakened by sharp V-notches [2,3]. Lazzarin et al. [4] compared two widely employed criteria based on energy considering V-notched components under pure Mode I loading; the first criterion is based on the local SED [5] while the second criterion is the so-called FFM approach. Two different formulations of the FFM criterion is available in the literature: the first formulation was proposed by Leguillon [6,7] and the second one was proposed by Carpinteri et al. [8]. The main aim of the current paper is to extend the previous comparison to the case of Mode II loading condition.

The FFM criteria have been recently extended to the mixed Mode I/II and Mode II loading conditions [9-11]. Instead, considering the SED criterion, a new formulation for estimating the control radius under in-plane shear loading is proposed here and is compared with results obtained from the expression which is valid for Mode I. Lazzarin and Zambardi [5] proposed the local SED criterion which is based on the strain energy density averaged over a control volume surrounding the notch tip. The size of control volume is dependent on the material property and the loading conditions [12,13]. The average SED value is independent of the mesh size [14]. Additionally, a new method for average SED calculations for cracks under mixed mode I+II loading by adopting very coarse meshes has been recently proposed [15,16] which is based on the maximum stresses obtained from FE analyses.
In this paper the analytical formulation of the three compared criteria [5,9-11] is introduced to obtain the critical Mode II Notch Stress Intensity Factor using different approaches. After all, the mentioned criteria are applied to sharp V-notched plates subjected to in-plane shear loading, in order to investigate the ability of each method to analysis the fracture behaviour of brittle materials. The experimental data available in the literature were used for the analyses [17].

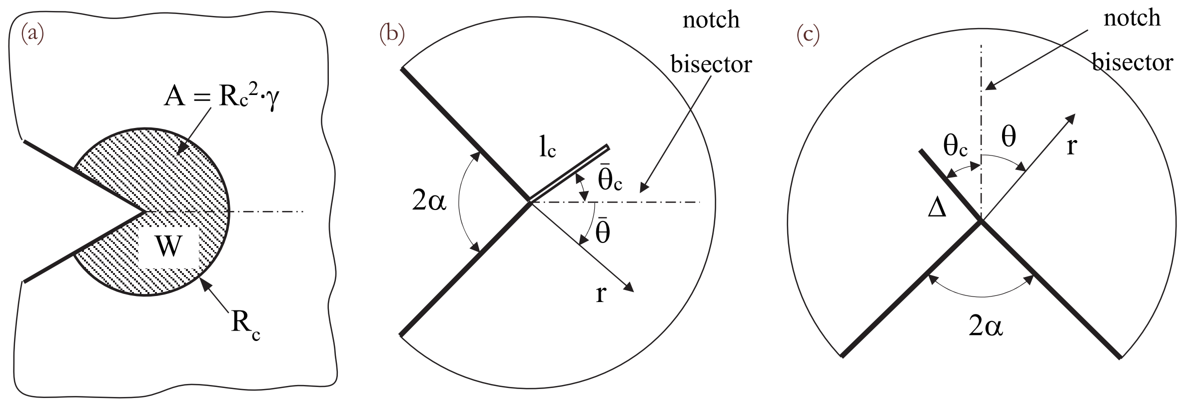

Figure 1: Reference system for: (a) averaged SED criterion; (b) Leguillon et al. criterion and (c) Carpinteri et al. criterion.

Failure criteria for sharp V-notches under pure Mode II loading

Averaged strain energy density (SED) criterion

According to Lazzarin and Zambardi [5], the fracture of a brittle material takes place when the strain energy density averaged over a control volume characterized by a radius R_c (Fig. 1a), becomes equal to the critical value W_c (Eq. 1). In the case of a smooth component under nominal shear loading condition, employing Beltrami's hypothesis, the following expression can be derived:

$$W_c = \frac{\tau_c^2}{2G} = \frac{(1+\nu)\tau_c^2}{E} \tag{1}$$

where τ_c is the ultimate shear strength, G the shear modulus and E the Young's modulus, while ν represents the Poisson's ratio. Considering a V-notched plate subjected to nominal pure Mode II loading, the relationship $\overline{W} = W_c$ is verified under critical conditions. Accordingly, one can obtain the expression for K_{2c}, which is the critical NSIF at failure:

$$\frac{e_2}{E}\frac{K_{2c}^2}{R_c^{2(1-\lambda_2)}} = \frac{(1+\nu)\tau_c^2}{E} \quad \Rightarrow \quad K_{2c} = \sqrt{\frac{(1+\nu)}{e_2}}\tau_c R_c^{(1-\lambda_2)} \tag{2}$$

The control radius R_c can be evaluated by considering a set of experimental data that provides the critical value of the Notch Stress Intensity Factor for a given notch opening angle. If the V-notch angle is equal to zero ($2a = 0$, $\lambda_2 = 0.5$), the case of a cracked specimen under nominal pure Mode II loading is considered, so that under critical conditions K_{2c} coincides with the Mode II fracture toughness K_{IIc}. Then, taking advantage of Eq. (2), with $K_{2c} \equiv K_{IIc}$, and following the same procedure proposed by Yosibash et al. [18] for obtaining the control radius under Mode I loading condition, the expression of R_c turns out to be:

$$R_{c,II} = \frac{e_2(2\alpha=0)}{(1+\nu)}\left(\frac{K_{IIc}}{\tau_c}\right)^2 = \frac{(1+\nu)(9-8\nu)}{8\pi(1+\nu)}\left(\frac{K_{IIc}}{\tau_c}\right)^2 = \frac{(9-8\nu)}{8\pi}\left(\frac{K_{IIc}}{\tau_c}\right)^2 \tag{3}$$

Moreover, it is useful to express the NSIF at failure K_{2c} as a function of the Mode I material properties (K_{Ic} and σ_c), which are simpler to determine or to find in the literature than Mode II material properties. For this purpose, it is possible to approximately estimate the Mode II fracture toughness (K_{IIc}) as a function of K_{Ic}, according for example to Richard et al. [19]. In the same manner, it is possible to approximately estimate the ultimate shear strength (τ_c) as a function of the tensile one (σ_c). With reference to brittle materials with linear elastic behaviour (as for example polymethylmethacrylate, graphite,…), it has been observed experimentally [20] that the most appropriate criterion is that of Galileo-Rankine. Accordingly the following expressions are valid:

$$K_{IIc} \cong \frac{\sqrt{3}}{2} K_{Ic} \tag{4}$$

$$\tau_c = \phi\sigma_c \qquad (\phi = 0.80 \sim 1.00 \text{ on an experimental basis}) \tag{5}$$

Finally, substitution of Eqs. (3)-(5) into Eq. (2) gives the NSIF at failure K_{2c} in a more useful form:

$$K_{2c} = \left[\sqrt{\frac{1+\nu}{e_2}}\left(\frac{3}{4}\frac{9-8\nu}{8\pi}\right)^{(1-\lambda_2)} \phi^{2\lambda_2-1}\right] K_{Ic}{}^{2(1-\lambda_2)} \sigma_c{}^{2\lambda_2-1} \tag{6}$$

It should be noted that the control radius R_c could be in principle different under Mode I and Mode II loading condition, this means that it depends on the material properties but also on the loading conditions.

Finite Fracture Mechanics: Leguillon et al. formulation

By using Leguillon et al. criterion, it is thought that at failure an incremental crack of length l_c initiates at the tip of the notch. According to Leguillon et al. [7,11], two conditions can be imposed on stress components and on strain energy and both are necessary for fracture. They have to be simultaneously satisfied to reach a sufficient condition for fracture.

On the basis of the stress condition, the failure of the notched element happens when the singular stress component normal to the fracture direction $\overline{\theta}_c$ is higher than the material tensile stress σ_c all along the crack of length l_c just prior to fracture.

On the basis of the condition imposed on strain energy, the failure occurs when the SERR $\widetilde{\mathcal{G}}$ reaches a value higher than $\mathcal{G}_c$, which is the critical value for the material. $\widetilde{\mathcal{G}}$ is the ratio between the potential energy variation at crack initiation (δW_p) and the new crack surface created (δS).

These two conditions can be formalised as follows providing a general criterion for the fracture of components in presence of pointed V-notches.

Stress criterion: $$\sigma_{\theta\theta}(l_c,\overline{\theta}_c) = k_2 l_c{}^{\lambda_2-1}\tilde{\sigma}^{(2)}_{\theta\theta}(\overline{\theta}_c) \ge \sigma_c \tag{7a}$$

Energy criterion: $$\widetilde{\mathcal{G}} = -\frac{\delta W_p}{\delta S} = \frac{k_2^2 H^*_{22}(2\alpha,\overline{\theta}_c) l_c{}^{2\lambda_2} d}{l_c d} \ge \mathcal{G}_c \tag{7b}$$

In Eqs. (7a,b) the length of the incremental crack is l_c (see Fig. 1b). λ_2 is the Mode II Williams' eigenvalues quantities [21], that is a function of the V-notch opening angle $2\alpha \tilde{\sigma}^{(2)}_{\theta\theta}(\overline{\theta})$ is a function of the angular coordinate $\overline{\theta}$, while d is the thickness of the notched element. Finally, $H^*_{22}(2\alpha,\overline{\theta}_c)$ is a "geometrical factor" function of the local geometry (2*a*) and of the fracture direction ($\overline{\theta}_c$).

Leguillon et al. criterion requires that conditions (7a) and (7b) must be simultaneously satisfied. The length of the incremental crack can be determined by solving the system of two equations (Eqs. (7a,b)), then by substituting it into Eq. (7a) or (7b), the fracture criterion can be expressed in the classical Irwin form ($K_I \geq K_{Ic}$). In this case the critical value of the NSIF k_{2c} can be provided as a function of the material properties (σ_c and $\mathcal{G}_c$), the V-notch angle 2*a* and the critical crack propagation angle $\overline{\theta}_c$.

$$k_2 \geq \left(\frac{\mathcal{G}_c}{H^*_{22}\left(2\alpha,\overline{\theta}_c\right)}\right)^{1-\lambda_2}\left(\frac{\sigma_c}{\tilde{\sigma}^{(2)}_{\theta\theta}\left(\overline{\theta}_c\right)}\right)^{2\lambda_2-1} = k_{2c} \tag{8}$$

Yosibash et al. [11] have computed the function H_{22} for a range of values of the notch opening angle 2*a* and of the fracture direction $\overline{\theta}_c$, taking into account a material characterized by a Young's modulus E = 1 MPa and a Poisson's ratio ν = 0.36. The function H^*_{22} for any other Young's modulus E and Poisson's ratio ν can be easily obtained according to the following expression:

$$H^*_{22}\left(2\alpha,\overline{\theta}_c\right)=H_{22}\left(2\alpha,\overline{\theta}_c\right)\frac{1-\nu^2}{E}\frac{1}{1-0.36^2} \tag{9}$$

A more useful expression for k_{2c}, as a function of the Mode I fracture toughness K_{Ic} and of the ultimate tensile stress σ_c, can be derived by substituting Eq. (9) and the link between $\mathcal{G}_c$ and K_{Ic} into Eq. (8). Then, by employing Gross and Mendelson's [22] definition for the critical NSIF K_{2c}, the following expression can be obtained:

$$K_{2c} = \left[\sqrt{2\pi}\left(\frac{1-0.36^2}{H_{22}\left(2\alpha,\overline{\theta}_c\right)}\right)^{1-\lambda_2}\left(\frac{1}{\tilde{\sigma}^{(2)}_{\theta\theta}\left(\overline{\theta}_c\right)}\right)^{2\lambda_2-1}\right]K_{Ic}^{\,2(1-\lambda_2)}\sigma_c^{\,2\lambda_2-1} \tag{10}$$

Finite Fracture Mechanics: Carpinteri et al. formulation

In a similar manner to Leguillon, a fracture criterion for brittle V-notched elements based on FFM concept has been proposed by Carpinteri et al. in [8-10]. Under critical conditions, a crack of length Δ is thought to initiate from the notch tip. Again, a sufficient condition for fracture can be achieved from the satisfaction of both a stress criterion and an energy-based one.

On the basis of the averaged stress criterion, the failure of the component at the V-notch tip happens when the singular stress component normal to the crack faces, averaged on the crack length Δ, becomes higher than the tensile stress σ_c of the material under investigation.

The energy-based condition, instead, requires for the failure to happen that the strain energy released at the initiation of a crack of length Δ is higher than the material critical value, which depends on $\mathcal{G}_c$. By considering the relationship between the SERR $\mathcal{G}$ and the SIFs K_I and K_{II} of a crack under local mixed mode I+II loading, it is possible to derive a more useful formulation. This is valid under plane strain hypotheses and considering that the crack propagates in a straight direction.

The contemporary verification of the conditions given by Eq. (11a) and (11b) allows to formalize a criterion for the brittle fracture of sharply V-notched elements:

Average stress criterion:
$$\int_0^\Delta \sigma_{\theta\theta}\left(r,\theta_c\right)dr = \int_0^\Delta \frac{K^*_{II}}{\left(2\pi r\right)^{1-\lambda_2}}\tilde{\sigma}^{(2)}_{\theta\theta}(\theta_c)\,dr \geq \sigma_c\Delta \tag{11a}$$

Energy criterion:
$$\int_0^\Delta -\frac{dW_p}{da}da = \int_0^\Delta \mathcal{G}\left(a,\theta_c\right)da \geq \mathcal{G}_c\Delta \tag{11b}$$

$$\int_0^{\Delta}\left[K_I^2\left(a,\theta_c\right)+K_{II}^2\left(a,\theta_c\right)\right]da=\int_0^{\Delta}\left[K_{II}^{*\,2}a^{2\lambda_2-1}\left(\beta_{12}{}^2\left(2\alpha,\theta_c\right)+\beta_{22}{}^2\left(2\alpha,\theta_c\right)\right)\right]da\geq K_{Ic}^2\Delta$$

In Eqs. (11a) and (11b), Δ represents the length of the crack initiated at the V-notch tip (see Fig. 1c), while λ_2 is the Mode II Williams' eigenvalue [21]. (r, θ) are the polar coordinate system centred at the notch tip and a represents a generic crack length.

With the aim to employ the energy-based approach, the knowledge of the SIFs K_I and K_{II} of the tilted crack nucleated at V-notch tip, as a function of the crack length a is strictly required. In order to this, the expressions of the SIFs K_I and K_{II} derived by Beghini et al. [23], on the basis of approximate analytical weight functions, can be used. They are functions of the crack length a, the V-notch angle 2α, the fracture direction θ_c and the NSIF K_{II}^*.

It is useful to introduce a simplified notation according to Eq. (12), in which the relationship between the parameter $\bar{\beta}_{22}$ according to Sapora et al. [10] and the parameter H_{22} according to Yosibash et al. [11] is shown, as highlighted also in [9].

$$\bar{\beta}_{22}\left(2\alpha,\theta\right)=\frac{\beta_{12}{}^2\left(2\alpha,\theta\right)+\beta_{22}{}^2\left(2\alpha,\theta\right)}{2\lambda_2}=\frac{H_{22}\left(2\alpha,\theta\right)\left(2\pi\right)^{2\lambda_2-2}}{1-0.36^2} \tag{12}$$

On the basis of Carpinteri et al. approach, the fracture of the component happens when both conditions provided by Eqs. (11a) and (11b) are simultaneously satisfied. By solving the system of two equations, the length of the initiated crack Δ and the critical NSIF K_{IIc}^* can be explicitly found. Moreover, by adopting the classical definition of K_{2c} due to Gross and Mendelson [22], the following expression results to be valid:

$$K_{2c}=\left[\left(2\pi\right)^{\left(\lambda_2-\frac{1}{2}\right)}\left(\frac{1}{\bar{\beta}_{22}\left(2\alpha,\theta_c\right)}\right)^{1-\lambda_2}\left(\frac{\lambda_2\left(2\pi\right)^{1-\lambda_2}}{\tilde{\sigma}_{\theta\theta}^{(2)}\left(\theta_c\right)}\right)^{2\lambda_2-1}\right]K_{Ic}{}^{2(1-\lambda_2)}\sigma_c{}^{2\lambda_2-1} \tag{13}$$

ANALYTICAL COMPARISON

The criteria taken into consideration in the present contribution, can be compared on the basis of the final relationships of the Mode II critical Notch Stress Intensity Factor according to Gross and Mendelson's definition [22], see Eqs. (6), (10) and (13). The same proportionality relation is common to all criteria:

$$K_{2c}\propto K_{Ic}{}^{2(1-\lambda_2)}\sigma_c{}^{2\lambda_2-1} \tag{14}$$

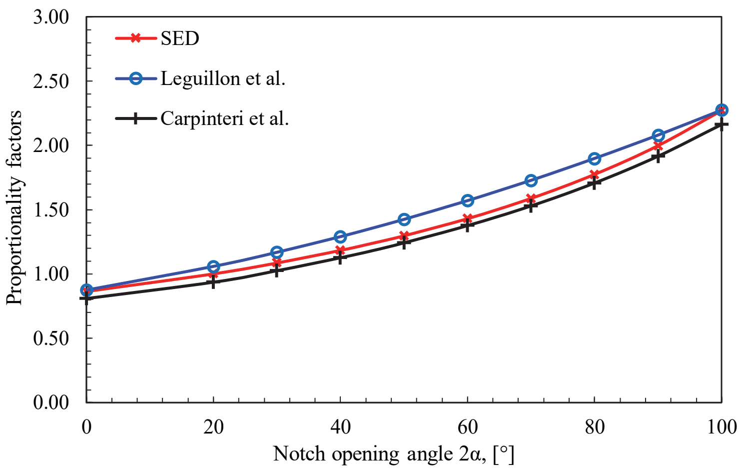

Figure 2: Comparison among proportionality factors given by Eqs. (6), (10) and (13).

The only difference in the final expressions is represented by the proportionality factor. Indeed, the latter is a function of the V-notch angle (2α) in Leguillon et al. and Carpinteri et al. formulations. On the other hand, the proportionality factor of the SED approach is a function also of the Poisson's ratio ν. It should be noted that the V-notch tip singularity ($1-\lambda_2$), tied to Mode II loading, disappears for opening angles higher than 102 degrees, according to Williams [21]. Therefore the analytical comparison of Eqs. (6), (10) and (13) has been carried out for notch opening angles 2α between 0 and about 100 degrees.
The proportionality factors are very similar in their trends (Fig. 2). In particular, the factor of Leguillon et al. approach is always a bit greater than those obtained by using the other two criteria. The factors according to Leguillon et al. and SED criteria assume the same values for 2α= 0 and 100 degrees: 0.87 and 2.27, respectively. The factor based on Carpinteri et al. approach, instead, does not match that of the other criteria for any value of 2α.

COMPARISON BASED ON EXPERIMENTAL DATA

The assessment capability of the considered criteria is compared here by employing experimental data reported in the literature. The data are all relevant to components weakened by sharp V-notches and constituted by polymethylmethacrylate (PMMA). The material properties and the experimental details are summarised in the following. A more extended comparison can be found in [15].
The experimental results and the theoretical predictions obtained from Eqs. (6), (10) and (13) have been compared on the basis of the critical value of the Mode II NSIF, K_{2c}. The critical NSIFs are plotted as a function of the notch angle 2α. If not explicitly given in the original contributions, the NSIFs to failure K_{2c} were calculated from 2D FE analyses, by adopting FE meshes consisting of eight node solid elements (PLANE 183) and by applying to the FE models the critical loads experimentally obtained. All FE analyses have been performed by means of ANSYS®, version 14.5. It should be noted that K_{2c} is coincident with the NSIF K_2 evaluated on the notch bisector line according to Gross and Mendelson definition [22] ($K_2 = \sigma_{r\theta}\sqrt{2\pi}r^{1-\lambda_2}$ with $r \to 0$), provided that the FE model is loaded with the experimental critical load.

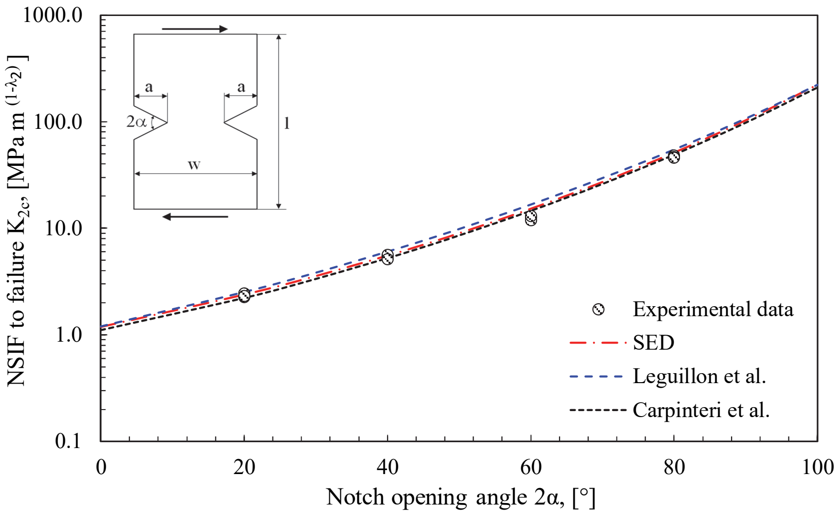

Figure 3: Plot of the NSIF to failure K_{2c} (Log-scale) and comparison with the experimental data from PMMA double V-notched specimens.

The considered set of experimental results has been derived from Arcan tests carried out by Seweryn et al. [3] on PMMA double V-notched specimens. The tested V-notched components were characterized by a length l = 200 mm, a width w = 100 mm, a notch depth a = 25 mm and a thickness t = 5 mm. Seweryn et al. [3] took into account specimens with four different V-notch angles, 2α = 20, 40, 60 and 80 degrees, being the loading angle to obtain pure Mode II loading equal to ψ = 90 degrees [24]. Three PMMA samples have been tested for each geometry. The relevant properties of the considered material have been reported in the original contribution [24] as follows: the Young's modulus E was equal to 3000 MPa, the Poisson's ratio equal to 0.30, while the Mode I fracture toughness and the critical tensile stress were K_{Ic} = 1.37 MPa.m$^{0.5}$ and σ_c = 115 MPa, respectively.

The experimental results relevant to the PMMA V-notched components are shown in Fig. 3 in terms of the critical NSIF K_{2c}, along with the theoretical predictions based on the fracture approaches under consideration. It can be observed from Fig. 3 that the agreement between theoretical predictions and experimental results, in terms of critical Mode II NSIF K_{2c}, is very good for all considered criteria. Some recent developments dealing with creep stresses are reported in [25].

CONCLUSIONS

Three different fracture criteria for brittle and quasi-brittle materials weakened by sharp V-notches have been considered in the present contribution. The attention has been focused on in-plane shear loading conditions (Mode II). The averaged strain energy density (SED) criterion and two different formulations of the Finite Fracture Mechanics (FFM) theory, according to Leguillon et al. and to Carpinteri et al. respectively, have been accurately compared. With reference to the criterion based on the averaged SED, a new expression for estimating the control radius R_c under pure Mode II loading has been proposed.

First, the criteria have been compared analytically by providing the expressions of the critical value of the Notch Stress Intensity Factor K_{2c}. The same proportionality relation has been found to exist between K_{2c} and two key material properties: the Mode I fracture toughness K_{Ic} and the ultimate tensile stress σ_c. The only difference between the analysed criteria is represented by the proportionality factor.

Finally, the approaches taken into consideration in the present contribution have been adopted for the fracture assessment of brittle V-notched components subjected to pure Mode II loading. This has allowed to investigate the assessment capability of each approach under in-plane shear loading. A set of experimental data reported in the literature has been employed for the comparison. The agreement between experimental data and theoretical predictions has been found very good for all criteria considered in the present investigation.

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