



Evaluation of mixed mode I/II fracture toughness of C 50/60 from Brazilian disc test

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ABSTRACT. Durability and sustainability of structures made from concrete like materials is getting more into an interest of civil engineers. Structures are subjected not only to uniaxial load, this means if there is a crack inside the load could be divided into mode I and shear mode II, but also to a mixed mode I/II. Therefore, it is necessary to perform test, which covers mixed mode loads and could be used for specimen made from concrete core-drill. Recommended test specimens used for evaluation of fracture mechanical parameters has usually a prismatic or rectangular shape. The cost of reshaping cylinder into rectangular is expensive and not very efficient, therefore it is very effective to use Brazilian disc test specimen with central notch to obtain mixed mode fracture parameters. The contribution deals with numerical support for Brazilian disc test to obtain calibration curves for mode I and mode II, evaluation of experimental results and compare them with data adapted from literature.

KEYWORDS. Fracture Mechanics; Fracture Toughness; Centrally Cracked Brazilian disc; Mixed Mode I/II; Concrete.



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INTRODUCTION

Today's trends in civil engineering pushes investors and owners of concrete structures more often into renovation, than into demolition of structures. This fact leads into need for knowledge of material characteristics such as bulk density, Young's modulus of elasticity [1], compressive strength [2], flexural strength [3], etc. Some types of civil engineering structures are subjected to mixed mode loading I and II, therefore the knowledge of fracture mechanical parameters is necessary to predict life-time of concrete structures (bridges, cooling towers). To obtain material sample from considered structure, a core-drill is used, which drills out a cylindrical sample from a structure. This specimen is tested to obtain mechanical properties, but not for fracture mechanical parameters.



Fracture mechanical parameters of cementitious materials are usually obtained from recommended tests such as: three-point (3PBT) and four-point (4PBT) bending test with notch in tested specimen [4], for mixed mode load [5], wedge splitting test (WST) [6-9], or a combination of WST/3PBT [10] and modified compact tension test (MCT) [11, 12]. All tests have a predefined prismatic geometry and using them on specimens made from the core-drill is expensive and not very efficient, therefore it is very appropriate to use a Brazilian disc test specimen with central notch (circle cut from the core-drill cylinder) [13-16] to determinate fracture parameters of building materials see Fig. 1.

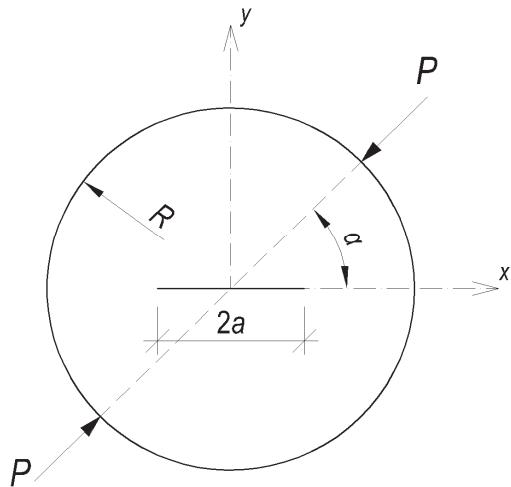


Figure 1: Geometry of a typical Brazilian disc specimen with load position alongside crack.

The main advantage of Brazilian disc is, that it could be used for investigation of fracture toughness for mode I, mode II and mixed mode by rotating the cracked against the load positions. This article compares the measured experimental data with data presented in [17].

THEORETICAL BACKGROUND

Mechanical properties

The unnotched Brazilian disc is very often used as an indirect test to determinate tensile strength of rock materials, therefore it is very valuable to use it to obtain tensile strength of concrete [18]. The tensile strength can be evaluated from following equation:

$$\sigma_t = \frac{2P}{\pi DB} \quad (1)$$

where σ_t is tensile stress, P is compressive load, D is diameter of disc and B is specimen's thickness.

Fracture Mechanics

This contribution is based on a linear elastic fracture mechanics. The linear elastic fracture mechanics concept uses the stress field in the close vicinity of the crack tip described by Williams's expansion [19]. This expansion is an infinite power series originally derived for a homogenous elastic isotropic cracked body, which can be described by a following equation:

$$\sigma_{ij} = \frac{K_I}{\sqrt{\pi r}} f_{ij}^I(\theta) + \frac{K_{II}}{\sqrt{\pi r}} f_{ij}^{II}(\theta) + O_{ij}(r, \theta) \quad (2)$$

where σ_{ij} represents the stress tensor components, K_I, K_{II} is the stress intensity factor for mode I respectively mode II, f_{ij}^I, f_{ij}^{II} , are known shape functions for mode I and mode II, O_{ij} represents higher order terms and r, θ are the polar coordinates (with origin at the crack tip; crack faces lie along the x -axis).



The values of the stress intensity factor (SIF) for a finite specimen and the polar angle $\theta = 0^\circ$ can be expressed in the following form [20-22]:

$$K_I = \frac{P\sqrt{a}}{RB\sqrt{\pi}} f_I(a/R, \alpha) \quad (3)$$

$$K_{II} = \frac{P\sqrt{a}}{RB\sqrt{\pi}} f_{II}(a/R, \alpha) \quad (4)$$

where P is compressive load, a is a crack length, R is radius of the disc ($D/2$), B is disc thickness and $f_I(a/R, \alpha)$, $f_{II}(a/R, \alpha)$ are dimensionless shape functions (calibration curves) for mode I and mode II, see Figs. 2 and 3.

NUMERICAL MODEL

Calibration curves

To obtain right calibration curves for each mode of SIF a numerical simulation was necessary. The numerical simulation was performed in finite element (FE) software Ansys 17.2 [23]. A two-dimensional (2D) numerical model with plane strain boundary condition were used to calculate SIF (K_I and K_{II}). The numerical model was meshed with element type PLANE183 taken from ANSYS's elements library and command KSCON was used to take into account the crack tip singularity. Input material properties of concrete used in FE software are following: Young's modulus and Possion's ratio, $E = 40$ GPa and $\nu = 0.2$, respectively. The geometry of disc has radius $R = 50$ mm and the relative crack length a/R varied from $<0.1; 0.9>$, notch angle α varied $<0^\circ; 90^\circ>$ and was loaded with constant force $P = 100$ N in all calculated cases.

In FE software ANSYS, the following equations are used for calculation of the SIF K_I and K_{II} for $\theta = \pm 180^\circ$.

$$K_I = \sqrt{2\pi} \frac{2G}{1+\kappa} \frac{|v|}{\sqrt{r}} \quad (5)$$

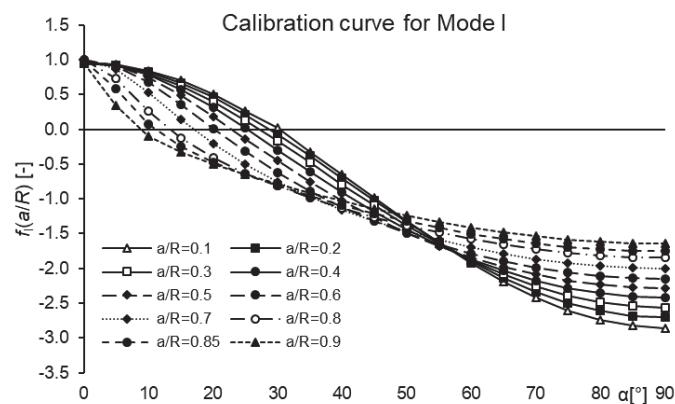
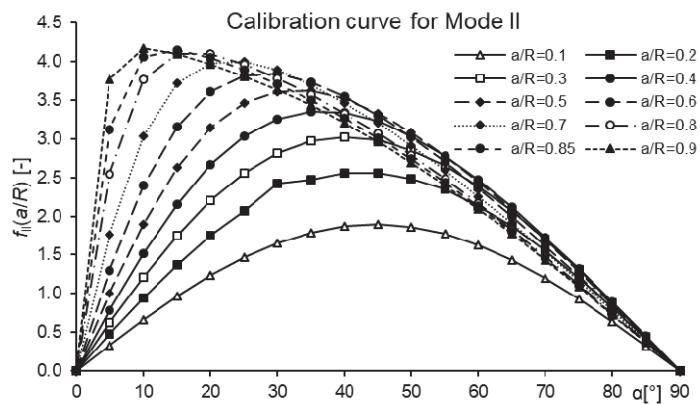
$$K_{II} = \sqrt{2\pi} \frac{2G}{1+\kappa} \frac{|u|}{\sqrt{r}} \quad (6)$$

where u , v are nodal displacements, G shear modulus, κ is Kolosov's constant for plane strain respectively plane stress conditions and r is coordinate in cylindrical coordinate system. From Brazilian disc geometry, mention above a calibration curves for mode I and II for various notch angle α and a/R ratio can be determined as a following functions [25, 26]:

$$f_I(a/R, \alpha) = \frac{K_I RB\sqrt{\pi}}{P\sqrt{a}} \sqrt{1 - \frac{a}{R}} \quad (7)$$

$$f_{II}(a/R, \alpha) = \frac{K_{II} RB\sqrt{\pi}}{P\sqrt{a}} \sqrt{1 - \frac{a}{R}} \quad (8)$$

From Fig. 2, it can be noticed, that for some angle α and a/R , the value of calibration curve for mode I equals zero ($f_I(a/R, \alpha) = 0$). This means, that there is only mode II (pure shear mode). Therefore, the fracture toughness for mode II could be evaluated.

Figure 2: Calibration curve $f_I(a/R, \alpha)$ for mode I.Figure 3: Calibration curve $f_{II}(a/R, \alpha)$ for mode II.

EXPERIMENT FOR MATERIAL C 50/60

Material

Material used in the experimental program was standard concrete C 50/60, maximum aggregate size was 8 mm.

Specimen's geometry

Brazilian disc specimens were prepared from standardized cylindrical specimens used for evaluation of cylindrical compressive strength of concrete [2]. Notches were prepared by using water jet cutter. The dimensions of specimens are introduced in Tabs. 1 and 2.

Specimen nmr.	Diameter D [mm]	Thickness B [mm]
04	150	28.59
05	150	30.52
06	150	29.90

Table 1: Dimensions of Brazilian disc specimens.

Specimen nmr.	$a/R [-]$	Diameter D [mm]	Thickness B [mm]	Notch length a [mm]
05_4	0.2667	150	31.83	40.06
09_4	0.2667	150	30.68	40.00
04_4	0.2667	150	31.65	40.09
09_6	0.4	150	29.93	60.17
01_6	0.4	150	31.44	60.92
04_6	0.4	150	30.97	60.15
07_6	0.4	150	31.17	59.94

Table 2: Dimensions of Brazilian disc specimens with a center notch.

Experimental procedure

The machine for tests was Seidner D7940 with maximum loading capacity 4 000 kN. The load rate was 0.01 kN/s. Brazilian disc specimens without notch were tested to obtain the tensile strength.

Brazilian disc specimens with the notch were tested under the selected angles inclined against loading positions see Tab. 3.

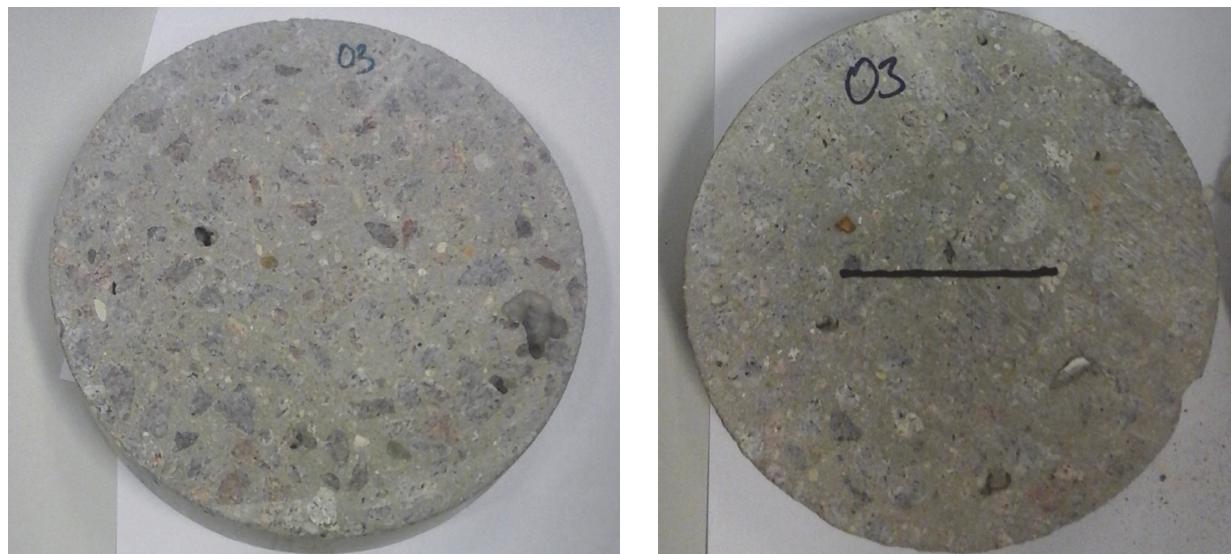


Figure 4: Example of specimens used for Brazilian disc test without and with notch

Specimen nmr.	$a/R [-]$	Inclination angle α [$^{\circ}$]
05_4	0.2667	0.0
09_4	0.2667	10.0
04_4	0.2667	27.2
09_6	0.4	0.0
01_6	0.4	0.0
04_6	0.4	10
07_6	0.4	25.2

Table 3: The relative length of notch in Brazilian disc specimens and angle of used angle position.



RESULTS AND DISCUSSION

Selected material and mechanical properties

Unnotched Brazilian disc specimens were subjected to compressive load to evaluate tensile strength from Eq. 1. The measured forces and calculated tensile strength are showed in Tab. 4.

Specimen nmr.	Measured force P [kN]	Tensile strength f_t [MPa]
04	34.9	5.18
05	38.8	5.39
06	35.7	5.06

Table 4: Measured forces and calculated tensile strength from eq. (1) for unnotched Brazilian disc.

The properties of used concrete are compared with material characteristics of mortar and concrete from Hou [17], for detailed information about composition of these materials see [17]. Comparison of selected mechanical properties is showed in Tab. 5.

	Bulk density ρ [kgm ⁻³]	Compressive strength f_c [MPa]	Tensile strength f_t [MPa]
Mortar from [17]	2018	25.56	3.6
Concrete [17]	2373	34.42	3.1
Concrete – present study	2321	55.4	5.21

Table 5: Comparison of selected mechanical characteristics of used concrete with data adapted from literature [17]

Fracture mechanical properties

Fracture mechanical properties of structural concrete were evaluated from Eq. (2) and (3). The values of stress intensity factors were evaluated for mode I (specimens 05_4,09_6 and 01_6), for mode II (specimens 04_4 and 07_6) and for mixed mode I/II. The evaluated fracture mechanical properties are summed up in Tab. 6.

Specimen nmr.	Measured force P [kN]	K_I [MPamm ^{1/2}]	K_{II} [MPamm ^{1/2}]
05_4	26.8	38.069	0.0006
09_4	24.3	30.513	25.906
04_4	25.2	0.0466	54.604
09_6	19.6	36.858	0.0012
01_6	20.0	35.784	0.0012
04_6	17.7	26.104	25.337
07_6	15.8	0.005	44.9745

Table 6: Measured experimental load P and calculated fracture mechanical parameters K_I and K_{II} for each specimen.

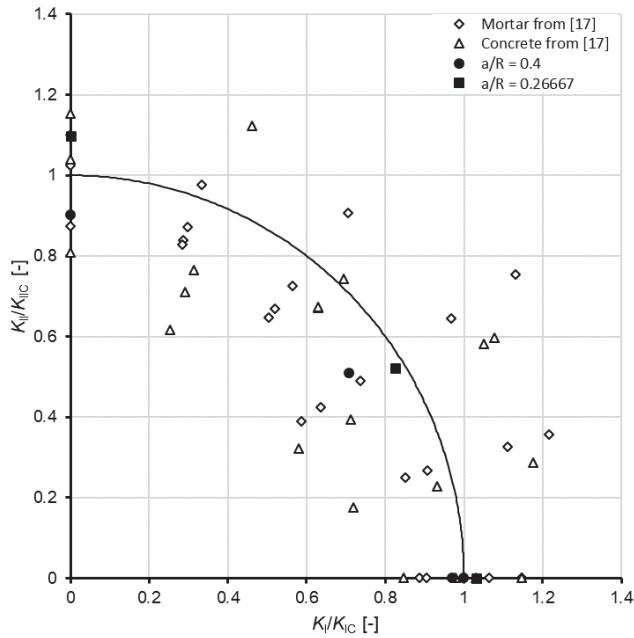


Figure 5: Comparison of fracture resistance K_{II}/K_{IC} (y axis) against K_I/K_{IC} (x axis) behavior with data from literature [17].

Comparison of experimental data

Experimental results from Hou [17] were digitalized and measured forces were used to be compared with data measured in the presented study. The comparison of results is done in two ways. The first one compares fracture resistance values of K_{II}/K_{IIC} (y axis) against K_I/K_{IC} (x axis) which is valuable to see pure mode I and pure mode II fracture resistance. An envelope curve (a circle with radius $K_I/K_{IC} = 1$ and $K_{II}/K_{IIC} = 1$) is plotted in Fig. 5. to see effect of mixed mode I/II failure for each tested specimen (different inclination angle α of notch).

The comparison made in Fig. 4. is not very often, therefore it is being necessary to make comparison which can be usually seen in literature. Fig. 5 compares normative values of K_{II}/K_{IC} (y axis) against K_I/K_{IC} (x axis) which is valuable to see mode II effects fracture resistance. An envelope curve (an ellipse with radius $K_{II}/K_{IC} = \text{mean value}$ and $K_I/K_{IC} = 1$) is plotted in Fig. 6. to see effect of mixed mode failure for each tested specimen (different inclination angle of notch).

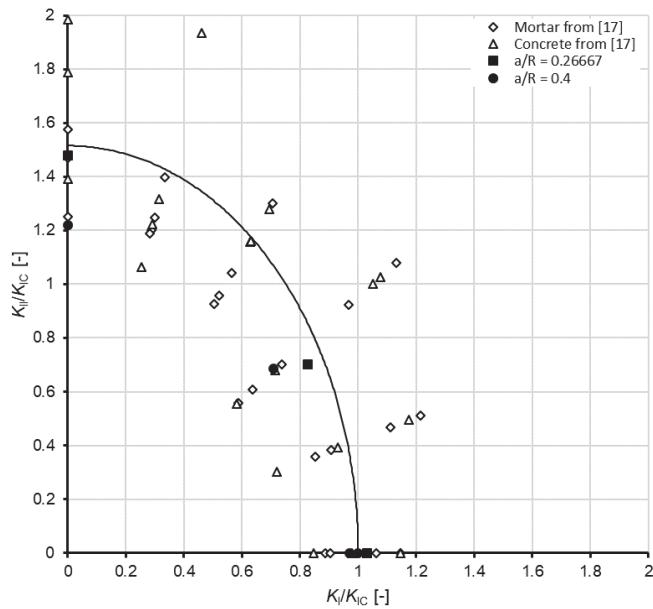


Figure 6: Comparison of fracture resistance K_{II}/K_{IC} (y axis) against K_I/K_{IC} (x axis) behavior with data from literature [17].



CONCLUSIONS

The present paper aims at analyzing the mixed mode fracture in a concrete C50/60 using the Brazilian disc test. A finite element analysis of Brazilian disc specimen, effects of crack angle rotation are studied based fracture mechanic approach. The following principal conclusions could be derived:

- The classical solutions of Linear Elasticity for the stress fields in the neighborhood of a crack tip in pure mode I or II can be obtained independently
- The solution for the stress field in a combined mode situation is obtained simply by rotation of a initiation crack.
- Experimentally obtain data for C 50/60 is compared with data from literature.

The presented numerical calculation and experimental results (C 50/60) will be used for future two-parameter fracture evaluation on alkali activated concrete [27, 28].

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