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Fatigue crack behaviour: comparing three-point bend test and wedge splitting test data on vibrated concrete using Paris' law

S. Seitl
Institute of Physics of Materials, Academy of Sciences of the Czech Republic, Zizkova 22, 616 62 Brno, and Brno University of Technology, Faculty of Civil Engineering, Veveří 331/95, Brno 602 00, Czech Republic
seitl@ipm.cz, http://orcid.org/0000-0002-4953-4324

T. Thienpont
Ghent University, Faculty of Engineering and Architecture, Valentin Vaerwyckweg 1, 9000 Ghent, Belgium and Brno University of Technology, Faculty of Civil Engineering, Veveří 331/95, Brno 602 00, Czech Republic

W. De Corte
Ghent University, Department of Structural Engineering, Faculty of Engineering and Architecture, Valentin Vaerwyckweg 1, 9000 Ghent, Belgium

ABSTRACT. The fatigue behaviour of concrete has become more important for the design of constructions due to the desire to build slimmer structures, which are more sensitive to fatigue loading. This article aims to evaluate and compare the fatigue crack propagation rate in vibrated concrete for four different stress ratios using the Paris-Erdogan law. The data evaluation in this article is based on crack mouth opening displacement (CMOD) measurements from cyclic three-point bending tests on single edge notched beams and from wedge splitting tests on notched cubes, obtained from experiments at Ghent University. For this study, finite element analysis is used to obtain a mathematical relationship between the CMOD and the relative crack length a/W, as well as a relationship between the stress intensity ratio ΔK and a/W. The obtained mathematical relationships were then combined with the measured CMOD values to correlate the test data to the Paris-Erdogan law. Herein, the crack propagation rate da/dN is plotted against the corresponding stress intensity range ΔK in a log-log graph. In a final step, the Paris-Erdogan law parameters C and m were obtained through linear curve fitting on the data points from the obtained graphs. The parameters C and m are then used to compare and evaluate the fatigue crack behavior in vibrated

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concrete, and the differences between the results from the three-point bend tests and wedge splitting tests.

KEYWORDS. Fatigue crack behaviour; Three-point bending test; Wedge splitting test; Self-compacting concrete; Paris-Erdogan law.

INTRODUCTION

Three-point bending tests (3PBT) as well as wedge splitting tests (WST) are often used to determine the fracture properties of structural materials such as cement based composites [8]. The material properties of normal, vibrated concrete (VC) and its behaviour under static loading are thoroughly researched and well known [9]. However, in order to reliably predict the behaviour of concrete in applications which involve millions of load cycles (e.g. bridges, beam cranes, offshore constructions), more research is required. For example, worldwide there are numerous concrete bridges that suffered excessive multi-decade deflections, some of which have already collapsed [2]. Fatigue behaviour in concrete is a complex process, and even though a tremendous effort has been made by the international scientific community, no universally accepted strategy suitable to efficiently perform the fatigue assessment of concrete has been agreed yet [4, 25, 27, 28].

This article aims to evaluate and compare the fatigue crack propagation rate in VC for different stress ratios using the Paris-Erdogan law [22]. The comparison is based on 3PBT and WST data for four different stress ratios, and was obtained during the research of Korte et al [11-14]. The test data was obtained from static tests (strength of material, fracture toughness, Young's modulus and Poisson ratio), and by performing cyclic tests on notched specimens, while measuring the crack mouth opening displacement (CMOD) for each load cycle. In this research, finite element analysis software ANSYS [1] was then used to correlate the measured CMOD data with the Paris-Erdogan crack propagation law [22, 27]. Herein, the crack propagation rate da/dN is plotted against the corresponding stress intensity range ΔK in a log-log graph. In a final step, the Paris law parameters C and m were obtained through linear curve fitting the data points from these obtained graphs. These parameters are then used to compare and evaluate the fatigue crack behaviour under the four stress ratios, as well as to compare the results from the three-point bend and the wedge splitting tests and data later could be used as input parameters for simulation e.g. ATENA [23, 24] or DOProC [15-16].

THEORETICAL BACKGROUND

Fatigue may be defined as a process of progressive, permanent internal structural changes in a material subjected to repeated loading. In concrete, these changes are mainly associated with the progressive growth of internal micro cracks, which results in a significant increase of irrecoverable damage [19]. Each load cycle induces microscopic cracks in the cement matrix, which gradually propagate during the loading process until an extended crack pattern is formed, leading to a significant change of the material properties [2].

In this article, the results of cyclic test on VC specimens, subjected to four different stress ratios are evaluated and discussed. In each test, the specimen was subjected to sinusoidal load function until failure, while measuring the CMOD at the crack mouth for each cycle, using a clip gauge. The 3PBT on single edge notched beams is a useful configuration for fracture toughness testing since it can be easily shaped and tested. For the test specimens, a value of $S/W=3$ was used in which S is the span between the supports, and W the depth of the specimen. Its geometry is included in all international standards for fracture toughness testing [7]. The WST on the other hand was first introduced by Linsbauer and Tschegg [20] and further developed by Brühwiler and Wittmann [5]. It is an interesting test setup, and just like the 3PBT, it can be performed using an ordinary electromechanical testing machine with a constant actuator displacement [26]. The geometry and measurements of the tested 3PBT and WST specimens are given in Fig.. 1.

In 1963, Paris and Erdogan proposed a very simple, yet highly useable relationship between the rate of crack propagation da/dN and the stress intensity range ΔK, expressed as [22]:

$$\frac{da}{dN} = C\Delta K^{m} \tag{1}$$

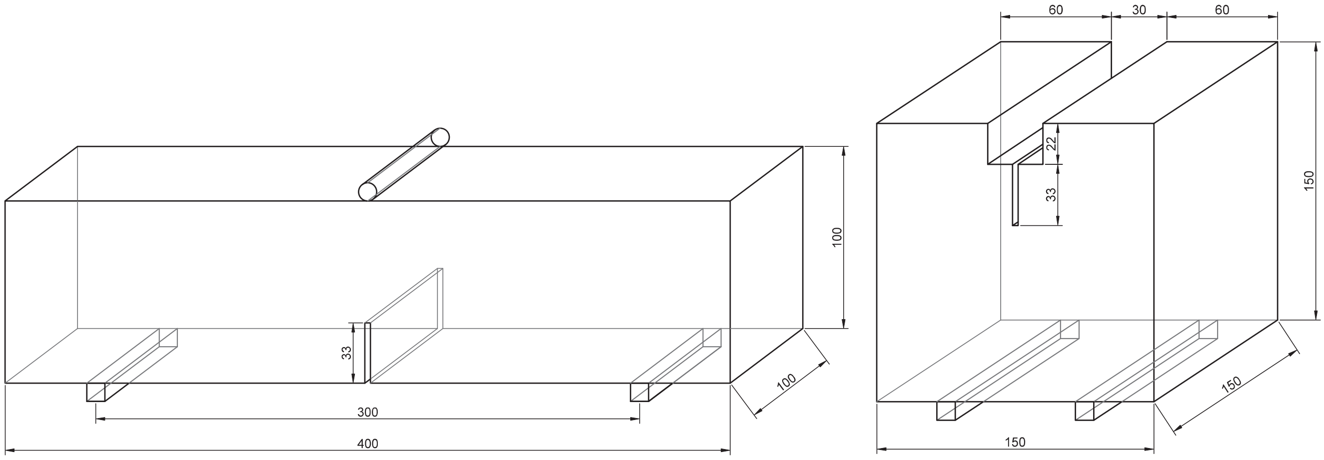

Figure 1: (a) 3PBT specimen geometry; (b) WST specimen geometry; (all units in mm).

Herein, *C* and *m* depend on the material, the specimen geometry and the loading conditions. They are therefore different for each material and must be obtained experimentally. The Paris-Erdogan law is applicable to a wide range of materials and describes their crack propagation behaviour in a relatively correct way over a wide range of stress ratios. If the crack propagation law for a certain material is known, it is possible to calculate by integration the number of cycles required for the crack to grow from one length to another [6]. In this article, the Paris' law parameters *C* and *m* will be used to compare the test data from both the 3PBT and WST.

Analysis in ANSYS

Numerical model

The finite element analysis software ANSYS [1] was used to create and evaluate various numerical 3PBT and WST models. These models were built using macro's in the ANSYS Parametric Design Language (APDL). For both geometries only one half of the test piece is modelled, since their shapes are symmetrical (Fig. 2). All calculations were executed as a simplified 2D model, using 8-node isoparametric PLANE183 elements. A comparative study was performed for the models of both geometries, in order to find a suitable mesh size which delivers results with great accuracy. For the 3PBT, four different mesh sizes were compared and it was concluded that a 1 mm mesh size is dense enough to obtain accurate results. Similarly, for the WST, a mesh size of 1.5 mm showed to be of great accuracy. In order to accurately model the stresses near the crack tip, the ANSYS command KSCON is used. This creates a dense circular around the crack tip and allows the calculation of the stress intensity factor, using the KCALC command. Since the differences in the results for the deflection and the stress fields for both 2D and a 3D models are very small [14, 21], using a 2D model is preferred. These simplified numerical models require little computing power compared to complex 3D models.

For all concrete mixtures, cyclic tests under four stress ratios *R* were executed in the research of Korte et al. In these stress ratios, the lower load limit of was chosen to be 10% of the average ultimate load of the static tests. For the upper limit various percentages were selected: 70%, 75%, 80%, and 90% [14]. The stress ratio *R* is usually expressed as:

$$R = \frac{\sigma_{min}}{\sigma_{max}} \tag{1}$$

Using this formula, the four stress ratios are defined as: $R_{10\text{-}70} = 0.1429$, $R_{10\text{-}75} = 0.1333$, $R_{10\text{-}80} = 0.1250$ and $R_{10\text{-}90} = 0.1111$. In order to calculate the crack propagation rate and stress intensity ranges for all ratios, the numerical model was loaded under 10%, 70%, 75%, 80% and 90% of the average ultimate load of the static tests. The material input parameters for concrete were taken from [14]: Young's modulus $E_{VC} = 38.4$ GPa, $E_{SCC1} = 38.1$ GPa, $E_{SCC2} = 35.3$ GPa and Poisson ratio $\nu_c = 0.2$. For the metal part in the numerical WST model, representing the roller bearing loading device, Young's modulus $E_s = 210$ GPa and Poisson ratio $\nu_s = 0.3$ were used.

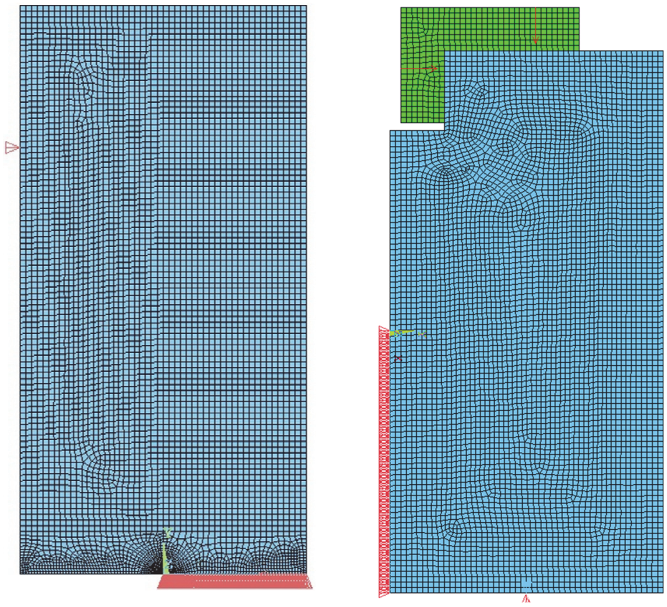

Figure 2: (a) 3PBT numerical model in ANSYS; (b) detailed view of the mesh near the crack tip.

Results

Neither da/dN nor ΔK, used in the Paris-Erdogan law, can be directly measured during a 3PBT or WST, and must therefore be obtained using a combination of finite element analysis and several calculation procedures. First, a mathematical relationship between the relative crack length α (= a/W), the dimensionless ratio between the crack length a and the specimen height W, and the CMOD was calculated through inverse analysis. This was achieved by calculating the CMOD for fixed values of a, at intervals of 0.1. In the WST, the relative crack length is defined as the vertical distance between the point where the splitting force is exerted and the support of the specimen. For a = 1, the total crack length a is therefore 145 mm; 5 mm shorter then the total WST specimen height. Fig. 3.a depicts the calculated CMOD values of the VC 3PBT geometry, under the four tested stress ratios, for α between 0.3 and 0.7. The plotted fitting curve (for VC 90%) shows an exponential relationship between the CMOD and the relative crack length can be found. Hence the great value for R^2 it can be stated that the exponential fitting curve is very accurate. Similar graphs for the WST can be obtained, also showing an exponential relationship between CMOD and α. Exponential fitting curves for the 3PBT and WST were calculated for all stress ratios. The inverse functions, which relate the relative crack length α to the CMOD where then used in further calculations.

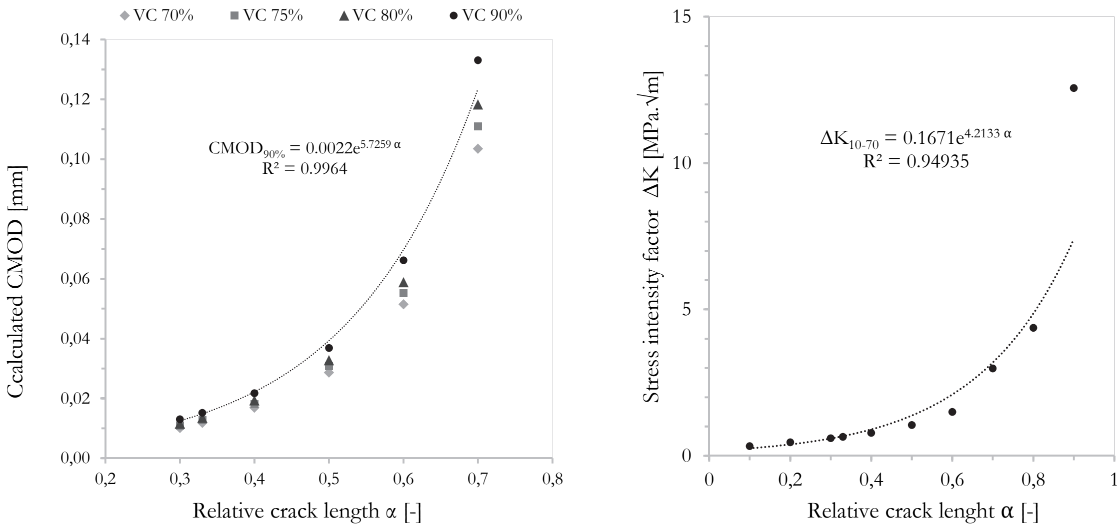

Figure 3: (a) CMOD calculations for different stress ratios; (b) ΔK calculations for 70% of the ultimate static load.

In a next step, the stress intensity range ΔK was computed for both geometries under the four stress ratios, using the built-in ANSYS command KCALC [1]. Fig. 3.b depicts the calculated value of ΔK from the 3PBT geometry for the 10-70% stress ratios. Similar to the CMOD calculations, a mathematical relationship between ΔK and α can be found through exponential curve fitting. Moreover, similar graphs for the WST can be obtained, also showing an exponential relationship between ΔK and α. The mathematical functions, which relate the relative crack length α to the stress intensity range ΔK where then used in further calculations.
In a final step, the crack propagation rate da/dN is plotted against the stress intensity ratio ΔK. As shown in Fig. 4, the data points with an according smaller value of ΔK don't fit the linear relationship described by the Paris-Erdogan law (grey colour). This is due to the fact that in concrete, two stages of crack growth can be observed: deceleration and acceleration [18]. Concrete fatigue fracture in the acceleration stage follows the Paris- Erdogan law [3][10]. Therefore, in order to obtain a fitting curve with a reasonably high R^2 value (index of determination), only the data points in the acceleration stage are used while the grey data points were ignored. This method was used to determine the linear fitting curves for all tested stress ratios.

DISCUSSION OF RESULTS

The data points and the linear fitting curves from the correlation of the 3PBT data are depicted in Fig. 5.a. The Paris' law parameters m and C in Eq. 1, which were obtained from the fitting curves of the da/dN – ΔK plots are given in Tab. 1. The last column of this table shows the number of load cycles N_{tot} from each test. For both the 10-80% and 10-90% stress ratio tests on the 3PBT samples, no results were found due to failure of the test specimen after only one or two load cycles.

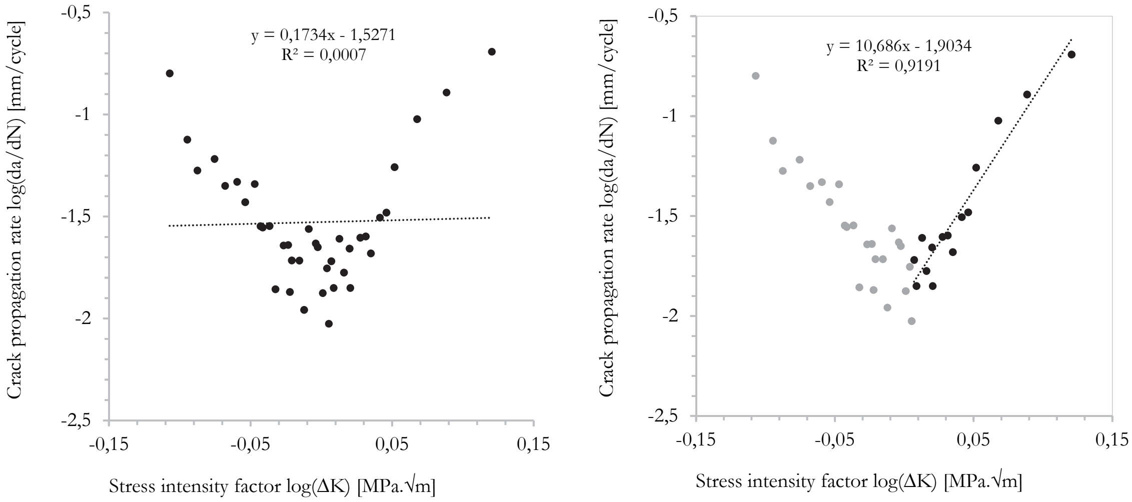

Figure 4: (a) Paris-Erdogan data points on log-log graph, showing a very poor result for the linear fitting curve; (b) data points in acceleration phase, with a reasonably accurate linear fitting curve.

From these results it can be concluded that the average value m_{avg} is greater for the 10-75% stress ratio compared to the 10-70% stress ratio. The difference is small however. This might be a consequence of the fact that the difference between 10-70% and 10-75% the stress ratios is rather small ($R_{10\text{-}70}$ = 0.1429 and $R_{10\text{-}75}$ = 0.1333). Despite the aforementioned it can be stated that when the value of ΔK increases, the crack propagation rate of for the 10-75% stress ratio increases faster compared to the 10-70% stress ratio. Based on the values of C_{avg}, no conclusions can be drawn.
Similar to the results from the 3PBT correlation, the results from the correlation of the WST data are given in Fig. 5.b and Tab. 2. From the WST's, no results were found for the 10-70% and 10-75% stress ratio. In the data from these tests, the crack length starts to decrease after approximately 40% of the total number of load cycles, resulting in negative values for da/dN, which cannot be plotted in a log-log graph. Therefore, no useful fitting curves were obtained for the aforementioned stress ratios. As a result, a comparison between the 3PBT and the WST based on this data is rather difficult.

Based on the values of m_{avg} no distinct difference can be observed between the 10-80% and 10-90% stress ratios. This might again be a consequence of the fact that the difference between 10-80% and 10-90% the stress ratios is small ($R_{10\text{-}80}$ = 0.1250 and $R_{10\text{-}90}$ = 0.1111). On the other hand, the average value C_{avg} is significantly greater for the 10-90% stress ratio compared to the 10-80% stress ratio. It can be concluded that for a fixed value of ΔK, the crack propagation rate for the 10-90% is considerably faster.

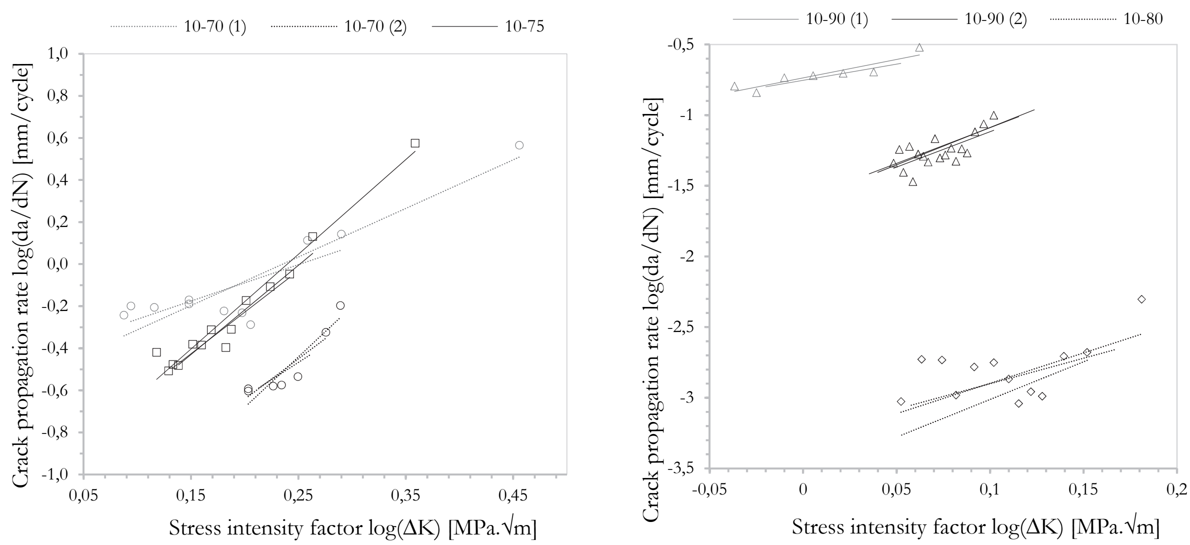

Figure 5: (a) Paris-Erdogan law fitting curves from correlation of 3PBT data from VC under 10-70% and 10-75% stress ratios; (b) Paris-Erdogan law fitting curves from correlation of WST data from VC under 10-80% and 10-90% stress ratios

Stress ratio	Test	Equation *m*.x+log(*C*)	*m*	m_{avg}	*C*	C_{avg}
10-70%	1	1.7301 x - 0.4349	1.7301	2.0198	0.6473	0.6141
		2.3094 x - 0.5433	2.3094		0.5808	
	2	3.2763 x - 1.2861	3.2763	3.9681	0.2763	0.2383
		3.8338 x - 1.4086	3.8338		0.2445	
		4.7943 x - 1.6396	4.7943		0.1941	
10-75%	1	4.2383 x - 1.065	4.2383	4.2299	0.3447	0.3558
		3.9467 x - 1.0177	3.9467		0.3614	
		4.5047 x - 1.0792	4.5047		0.3613	

Table 1: 3PBT – Fitting curve equations and Paris-Erdogan law parameters.

Stress ratio	Test	Equation *m*.x+log(*C*)	*m*	m_{avg}	*C*	C_{avg}
10-80%	1	3.6086 x - 3.2615	3.6086	4.4102	0.0383	0.0344
		4.2947 x - 3.3270	4.2947		0.0359	
		5.3274 x - 3.5448	5.3274		0.0289	
10-90%	1	2.2933 x - 0.7533	2.2933	2.4518	0.4708	0.4748
		2.6103 x - 0.7364	2.6103		0.4788	
	2	5.2891 x - 1.6165	5.2891	5.1182	0.1986	0.2000
		5.0523 x - 1.5937	5.0523		0.2032	
		5.0132 x - 1.6181	5.0132		0.1983	

Table 2: WST – Fitting curve equations and Paris-Erdogan law parameters.

Conclusions

In this contribution, the effect of the stress ratios on vibrated concrete was numerically studied, based on test results from 3PBT and WST samples. Despite the absence of data for certain stress ratios, the following conclusions can be drawn from this study:

- As a general conclusion from the 3PBT's and the WST's, it can be stated that both tests can be used to obtain valuable information about the fatigue crack propagation properties of both vibrated concrete and self-compacting concrete. For small stress ratios like 10-70% the 3PBT is more useful since it usually does not require more than 1000 load cycles until the test specimen fails. For higher stress ratios on the other hand, the WST is more useful, since for these higher stress ratios, the 3PBT specimens tend to fail after very few load cycles.
- In general, it can be concluded that the crack propagation in vibrated concrete is faster when the specimen is subjected to a higher stress ratio. A higher stress ratio results in a lower number of load cycles until failure and on average in larger values of the Paris' law parameters m and C.

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