Fatigue life assessment of thin-walled welded joints under non-proportional load-time histories by the shear stress rate integral approach

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ABSTRACT. Fatigue life tests under constant and variable amplitude loadings were performed on the tube-tube thin-walled welded specimens made of magnesium (AZ31 and AZ61) alloys. The tests included pure axial, pure torsional and combined in-phase and out-of-phase loadings with the load ratio $"R", ("\bar{R}") = -1$. For the tests with variable amplitude loads a Gauß-distributed loading spectrum with $L_S = 5 \cdot 10^4$ cycles was used.

Since magnesium welds show a fatigue life reduction under out-of-phase loads, a stress-based method, which takes this behavior into account, is proposed. The out-of-phase loading results in rotating shear stress vectors in the section planes, which are not orthogonal to the surface. This fact is used in order to provide an out-of-phase measure of the load. This measure is computed as an area covered by the shear stress vectors in all planes over a certain time interval, its computation involves the shear stress and the shear stress rate vectors in the individual planes. Fatigue life evaluation for the variable amplitudes loadings is performed using the Palmgren-Miner linear damage accumulation, whereas the total damage of every cycle is split up into two components: the amplitude component and the out-of-phase component. In order to compute the two components a modification of the rainflow counting method, which keeps track of the time intervals, where the cycles occur, must be used.

The proposed method also takes into account different slopes of the pure axial and the pure torsional Wöhler-line by means of a Wöhler-line

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interpolation for combined loadings.

KEYWORDS. Magnesium welds; Multiaxial fatigue; Variable amplitude loading; Out-of-phase loading; Rainflow; Fatigue life assessment.

INTRODUCTION

Fatigue life assessment of materials and components under multiaxial cyclic loading is a complex task, which requires to take a number of phenomena into account, e.g.:

- Depending on the material state out-of-phase loadings can lead to a reduced or to an increased fatigue life compared to in-phase loadings [1];
- Different slopes und knee points of the Wöhler-lines under pure axial and pure torsional loadings [2,3];
- Influence of mean stresses [4].

Magnesium thin-walled welds exhibit a fatigue life reduction under non-proportional loadings as well as different slopes of the Wöhler-lines for pure axial and pure torsional loads. The influence of mean stresses is not considered in this paper.
Fatigue life evaluation is often performed using stress-based methods because of their simplicity and convenience. Conventional stress-based methods, such as described in [5-7] were developed in order to calculate the so-called [8] fatigue limit. They usually lack the capability to assess the fatigue life reduction under out-of-phase loadings correctly. There is a number of more advanced methods, which take the out-of-phase behavior of the load into account explicitly, usually using an out-of-phase factor of some sort [9-13]. With these out-of-phase factors the stresses are scaled so, that the correct fatigue life is computed for an out-of-phase loading.
Another approach is employed by the IDD-Method [14]: no equivalent stress value is computed, however the damage of a load-time history or its part is computed according to the formula:

$$D_{total}^2 = D_{in-phase}^2 + D_{out-of-phase}^2 \tag{1}$$

where $D_{in-phase}$ and $D_{out-of-phase}$ are the IDD-specific damages resulting from the amplitude values and from the out-of-phase behavior of the loading. In fact in [14] there are two types of the out-of-phase damage: the one which is associated with the rotating principal stress axes and the one associated with out-of-phase principal stress values and no rotation of the principal axes. A very similar approach is used in the current paper, however out-of-phase loadings with and without rotating principal stresses are characterized mathematically in a uniform way using a stress rate integral. The proposed method requires to consider a general (three-dimensional) stress state.
Many methods are capable to take different slopes of the pure axial and pure torsional Wöhler-lines into account, usually they contain a parameter, which, if dependency of the number of cycles N is introduced, allows to "pull" the axial and torsion Wöhler-lines together. In this case iterative methods are required in order to compute the equivalent stress and hence the fatigue life. A method to avoid the iterative computation is to use a Wöhler-line interpolation, as presented e.g. in [3]. In order to perform Wöhler-line interpolation a numerical value, which characterizes, if the load is pure axial, pure torsional or a combined one, is required.
Fatigue life evaluation under variable amplitude loadings requires a procedure, which identifies damaging events ("cycles"). In [3, 15] it is proposed to find the plane with the highest shear stress aplitude using the Maximum Variance Method (MVM) and perform the rainflow cycle counting. The 4-point rainflow cycle counting procedure [16, 17] was extended in order to keep track of the time intervals, in which each counting cycle occurred.
The method discussed in this paper requires calculation of the local stress components, these are obtained using the FE-modeling according to the fictitious radius approach for thin-walled welds with $r_{ref} = 0.05$ mm [2].

A SHORT OVERVIEW OF THE SPECIMENS AND EXPERIMENTAL RESULTS

Laserbeam-welded tube-tube specimens (specimen geometry is shown in Fig. 1) were tested in a servohydraulic testing rig with two actuators, so cyclic axial and torsional loadings can be induced independently from each other. The welds were made of two self-hardening magnesium alloys AZ31 and AZ61. The testing program included

Wöhler-tests under pure axial, pure torsional and combined in-phase and out-of-phase loadings and Gaßnertests using a Gauß-distributed amplitude spectrum with $L_S = 5 \cdot 10^4$ cycles.

The load ratio $R = -1$ for Wöhler-tests or $\overline{R} = -1$ for Gaßner-tests was applied. Linear-elastic FE-models of the specimens were created in order to obtain the local stress components. The weld was modelled according to the fictitious notch approach with $r_{ref} = 0{,}05$ mm [2]. For both materials a fatigue strength reduction under out-of-phase loading was observed. The results of Wöhler- and Gaßner-tests for AZ31 are presented in Fig. 1. The Wöhler-lines were obtained in [18]. For Gaßner-tests real damage sums D_{real} lie between 0.2 and 0.8 for pure axial, pure torsional and combined out-of-phase loadings and for combined in-phase loadings between 1.0 and 1.7 [19].

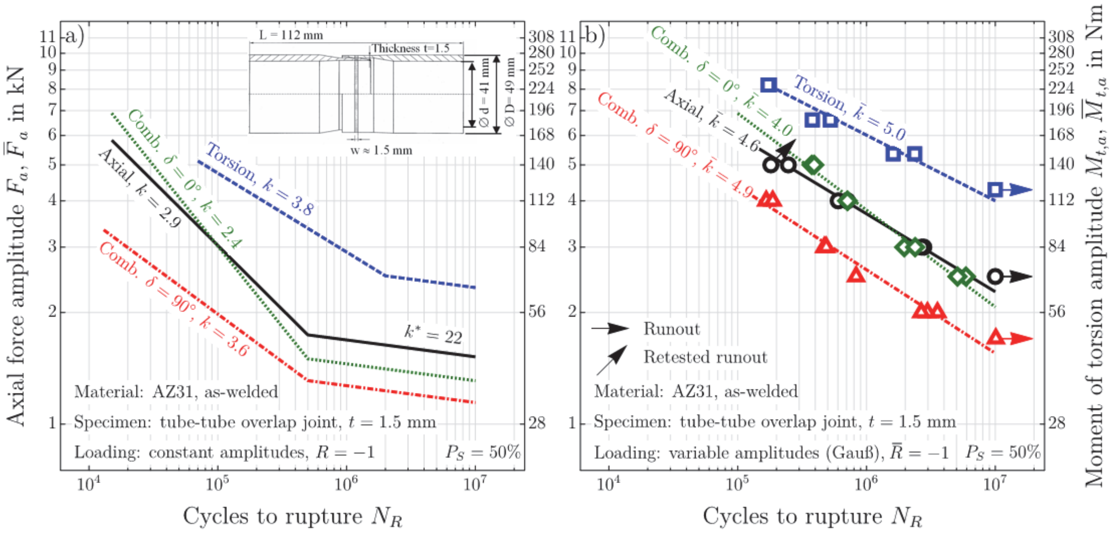

Figure 1: Wöhler (a) and Gaßner (b)-lines for the welded joints made of AZ31 alloy. The specimen geometry is shown as well.

SHEAR STRESS RATE INTEGRAL AS A MEASURE OF NON-PROPORTIONALITY

Non-proportionality of the loading is observed if the proportion between single load components changes over time. Non-proportionality can result from different configurations of the loading and can be accompanied by certain phenomena, like rotating principal directions of the stress tensor and/or presence of the mean stresses.

$$\boldsymbol{\sigma}(t) = \boldsymbol{\sigma}_0(t) + \overline{\boldsymbol{\sigma}} \tag{2}$$

Where $\overline{\boldsymbol{\sigma}}$ is the mean value of the tensor $\boldsymbol{\sigma}(t)$ over the time interval $[0,T]$ and $\boldsymbol{\sigma}_0(t) = \boldsymbol{\sigma}(t) - \overline{\boldsymbol{\sigma}}$, that is $\overline{\boldsymbol{\sigma}}_0 = 0$. In the absence of the mean stresses, i.e. if $\overline{\boldsymbol{\sigma}} = 0$, the non-proportionality can be observed, if the components of $\boldsymbol{\sigma}(t) = \boldsymbol{\sigma}_0(t)$ are out-of-phase. Thereby the non-proportionality can be accompanied by rotating principal stress directions, if the normal and the shear stress components are out-of-phase (or, speaking more generally, are uncorrelated), as well as by fixed principal directions, if the normal stress components are out-of-phase and the shear stress components are all zero. However both out-of-phase cases result in rotating shear stress vectors in the cross-section planes. The idea is to use this rotation of the shear stress vectors in order to characterize the out-of-phase behaviour of the loading. In a single plane given by the normal unit vector $\boldsymbol{n}$ the area $dA_{\boldsymbol{n}}$, which is covered by the movement of the time-dependent shear stress vector $\boldsymbol{\tau}_{\boldsymbol{n}}(t)$ in the infinitesimal time interval $[t, t+dt]$ computes to $dA_{\boldsymbol{n}} = \frac{1}{2}\left\|\boldsymbol{\tau}_{\boldsymbol{n}}(t) \times \dot{\boldsymbol{\tau}}_{\boldsymbol{n}}(t)\right\| dt$ (Fig.2), where the shear stress rate vector $\dot{\boldsymbol{\tau}}_{\boldsymbol{n}}$ is the time-derivative of $\boldsymbol{\tau}_{\boldsymbol{n}}$.

Now the out-of-phase behaviour in the infinitesimal $[t, t+dt]$ interval can be characterized as the "sum" of the areas $dA_{\boldsymbol{n}}$ in all the cross section planes associated with the different normal unit vectors $\boldsymbol{n}$:

$$dF_{OP} = \int_{\boldsymbol{n}} dA_{\boldsymbol{n}}\, d\boldsymbol{n} = \frac{1}{2}\left(\int_{\boldsymbol{n}} \left\| \boldsymbol{\tau}_{\boldsymbol{n}}(t) \times \dot{\boldsymbol{\tau}}_{\boldsymbol{n}}(t) \right\| d\boldsymbol{n}\right) dt.$$

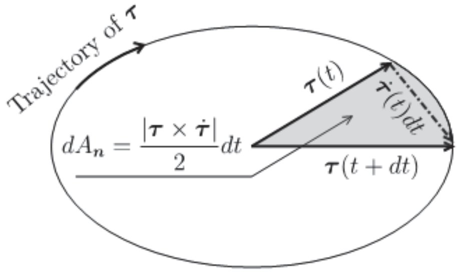

Figure 2: Trajectory of the shear stress vector in a given plane and the area $dA_{\boldsymbol{n}}$.

For a finite time interval $[t_1, t_2]$ the overall out-of-phase measure computes to:

$$F_{OP} = \frac{1}{2}\int_{t_1}^{t_2} \int_{\boldsymbol{n}} \left\| \boldsymbol{\tau}_{\boldsymbol{n}}(t) \times \dot{\boldsymbol{\tau}}_{\boldsymbol{n}}(t) \right\| d\boldsymbol{n}\, dt \;. \tag{3}$$

Formula (3) allows to express the out-of-phase behaviour of the loading in an arbitrary time interval, however it is dependent on the shear stress amplitudes and can attain any value between 0 and $+\infty$. Therefore, it is different from many out-of-phase measures, which can be found in the literature and attain values in the interval $[0,1]$, e.g. [9-12].

MAXIMUM NORMAL AND SHEAR STRESS AMPLITUDES AND WÖHLER-LINE INTERPOLATION

As described in [3, 15] the Maximum Variance Method (MVM) can be used in order to determine the plane, the direction and the value of the maximum shear stress amplitude $\tau_{a,max}$, it can also be applied in the same manner in order to determine the value of the maximum normal stress amplitude $\sigma_{a,max}$ as well as the plane, where it occurs. These two values can be computed for an arbitrary time interval and an arbitrary time-dependent stress history defined over this interval. For a pure torsional loading it follows $\tau_{a,max} = \sigma_{a,max}$ and for a pure axial loading $\tau_{a,max} = \frac{1}{2}\sigma_{a,max}$. Condition $\tau_{a,max} = 0$ corresponds to a hydrostatic cyclic loading. Whether there exists a cyclic loading with $\tau_{a,max} > \sigma_{a,max}$, is not known to the authors. If such loading exists, it should be a non-proportional one. The ratio $f_I = \frac{\tau_{a,max}}{\sigma_{a,max}}$ can be used to interpolate a Wöhler-line for a cyclic loading in the region between the pure axial loading and pure torsion, that is $\frac{1}{2} \le f_I \le 1$, as follows:

$$X = 2\left(X_{ax}\left(1 - f_I\right) + X_{tors}\left(f_I - \frac{1}{2}\right)\right). \tag{4}$$

Where X is a Wöhler-line parameter k, N_k or L_k, furthermore k, k_{ax}, k_{tors} represent the slopes of the respective Wöhler-lines, $N_k, N_{k,ax}, N_{k,tors}$ represent the number of cycles and $L_k, L_{k,ax}, L_{k,tors}$ the load amplitudes at the knee point. The values $L_k, L_{k,ax}, L_{k,tors}$ must refer to the same damage parameter, for instance v. Mises equivalent stress amplitude, the maximum normal stress amplitude, the maximum shear stress amplitude or any other equivalent stress amplitude. For the values $f_I \leq \frac{1}{2}$ the axial Wöhler-line is assumed, for $f_I \geq 1$ (if it can really occur) the torsional Wöhler-line. If an experimental Wöhler-line outside of the region between pure axial loading and pure torsion is available, the interpolation can be easily adapted to take it into account.

MODIFIED RAINFLOW COUNTING METHOD

Fatigue life under variable amplitude loadings requires an identification of damaging events (loading cycles, hysteresis loops, etc. depending on the particular setting), which is usually followed by a suitable damage accumulation procedure. One or another version of the rainflow cycling counting method, e.g. [16, 17, 20], is a commonly used procedure.

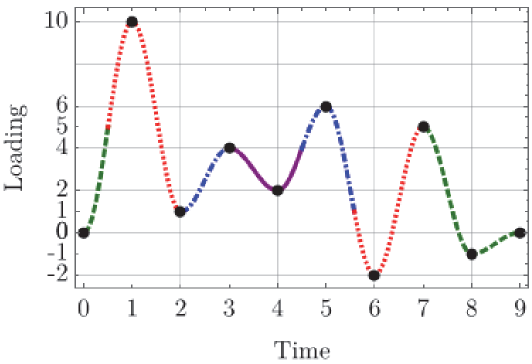

Figure 3: Schematic representation of the 4-point rainflow counting algorithm, that keeps time intervals in which cycles occur. Parts of the time-history of the same colour and dashing belong to a single cycle.

Damage accumulation is often performed using the Palmgren-Miner approach. In the multiaxial fatigue problems it is hard to define a cycle or a value, which is going to be counted. Some examples of application of the cycles counting procedures for multiaxial cyclic loadings can be found in [21-23]. These methods identify the cycles, but the information on the stress-time history is lost and hence the methods described in the previous two sections, especially Eq. 3, cannot be applied.

The following approach is employed in this paper: analogous to the MWC-Method [3] the plane and direction of the maximum shear stress amplitude is identified using MVM. The 4-point rainflow counting with subsequent residuum evaluation is performed. For each identified cycle the intervals in the stress-time history, over which it is distributed, are identified as well. So a cycle is given by a set of time intervals $\{I_1, I_2, \ldots, I_n\}$ in which it occurs. The number n of the intervals can vary between 1 and $+\infty$ depending on the load-time history. This approach is schematically shown in Fig. 3.

PUTTING IT ALL TOGETHER

The aim of the presented method is to evaluate fatigue life under cyclic variable amplitude in-phase and out-of-phase loadings ("arbitrary loadings"). The evaluation can be split in four steps. The required input data consist of: Wöhler-lines for a pure axial loading and pure torsion as well as a Wöhler-line for an out-of-phase loading. The Wöhler-lines must be provided in terms of some local equivalent stress amplitude. All three Wöhler-lines can be directly obtained from experiments or taken from the literature or some experience-based assumptions can be made.

Furthermore, a damage accumulation method as well as the damage sum must be defined.

The first step is to define the equivalent stress amplitude. Based on the position of the pure axial relative to the combined in-phase Wöhler-line (Fig. 1), the integral of the normal stress amplitudes over all plane is chosen. It is defined as follows:

$$\sigma_{a,eq} = \int_{\boldsymbol{n}} \sqrt{2Var(\sigma(\boldsymbol{n}))}\, d\boldsymbol{n}.$$

The second step is to compute the damage resulting from the out-of-phase loading. For the out-of-phase loading represented by the input out-of-phase Wöhler-line the values $f_{I,OP}$ and F_{OP} (Eq. 3) are computed, whereas the value $F_{OP}\left(\sigma_{eq,a}\right)$ depends on the equivalent stress amplitude. The value $f_{I,OP}$ allows to compute the interpolated Wöhler-line, which has the slope $k_{I,OP}$. In the case of magnesium welds it lies above the experimental out-of-phase Wöhler-line due to fatigue life reduction under out-of-phase loading. Now for each amplitude the total damage $D_{Total}\left(\sigma_{a,eq}\right)$ can be computed using the experimental Wöhler-line and the in-phase damage $D_{IF}\left(\sigma_{a,eq}\right)$ using the interpolated Wöhler-line. The additional damage due to out-of-phase behaviour is then given by:

$$D_{OF}\left(F_{OP}\left(\sigma_{eq,a}\right)\right)=\sqrt{D_{Total}^{2}\left(\sigma_{eq,a}\right)-D_{IF}^{2}\left(\sigma_{eq,a}\right)}. \tag{5}$$

The value (5) depends on the equivalent stress amplitude $\sigma_{eq,a}$ and since there is a one-to-one correspondence between $\sigma_{a,eq}$ and F_{OF}, D_{OF} is a function of F_{OF} and can be represented in the double logarithmic $F_{OF}-D_{OF}$ diagram, which is somewhat similar to the Wöhler-diagram.

The third step is the rainflow counting and identification of cycles and intervals in which these cycles occur.

In *the fourth step* for each cycle C the equivalent stress amplitude $\sigma_{a,eq,C}$, the value $f_{I,C}$, the interpolated Wöhler-line with the slope $k_{I,C}$ and the out-of-phase stress rate integral $F_{OF,C}$ are computed. Now the damages $D_{IF,C}\left(\sigma_{a,eq,C}\right)$ and $D_{OF,C}\left(F_{OF,C}\right)$ can be calculated. The total damage of the cycle computes to

$$D_{Total,C}=\sqrt{D_{IF,C}^{2}+\left(D_{IF,C}^{\frac{k_{I,C}}{k_{I,OP}}}\right)^{2}}.$$

The total damage of the loading sequence is then computed using Palmgren-Miner linear damage accumulation with modification according to Haibach with $k'=2k-2$ [24]: $D_{Total}=\sum_{C} D_{Total,C}$ and finally correcting the theoretical damage sum $D_{th}=1.0$ by the allowable one $D_{al}<1.0$ based on the experimental observation [25] the fatigue life computes to $N=\frac{L_S}{D_{Total}}D_{al}$.

FATIGUE LIFE EVALUATION RESULTS AND CONCLUSIONS

A stress-based fatigue life evaluation method, which is capable to process arbitrary multiaxial cyclic loadings, is proposed. It is capable to take into account effects like fatigue life reduction as well as fatigue life increase under out-of-phase loading. The method was applied in order to evaluate fatigue life of magnesium welds under multiaxial variable amplitude fatigue loadings. The results are shown in Fig. 4. The method can be extended in order to handle the mean stresses. In each plane the mean normal stress as well as the mean shear stress can be defined. A suitable way to take these values into account is yet to be defined.

For magnesium welds the integral of the normal stresses is chosen to be the equivalent stress in order to take into account the fact that combined in-phase Wöhler-line lies very close to the pure axial Wöhler-line. If necessary other effects can be taken into account, if other values of equivalent stress are chosen. For instance the maximum shear stress or the maximum normal stress amplitude will predict a fatigue life reduction under in-phase combined loading compared to the pure axial loading.

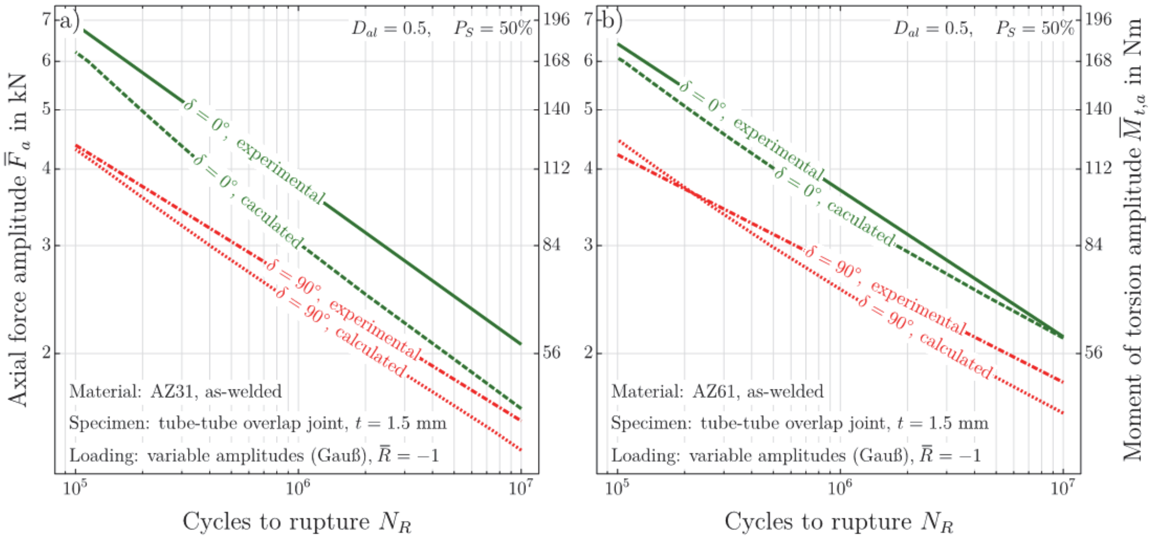

Figure 4: Computational and experimental Gaßner-lines for the welded joints made of AZ31 (a) and AZ61 (b) alloys, $D_{al} = 0.5$.

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