



Estimation of fatigue strength under multiaxial cyclic loading by varying the critical plane orientation

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ABSTRACT. The main purpose of this paper is to examine the influence of the critical plane orientation on the estimated fatigue strength of metals under multiaxial loading. The algorithm employed to evaluate fatigue strength implements the criterion of maximum normal and shear stress on a suitable damage plane (critical plane). The angle β defining the critical plane orientation is measured with respect to the direction that maximises the applied normal stress. Eleven (11) structural materials under combined bending and torsion cyclic loading are examined. For each analysed material, the value of β angle is selected so that the value of the scatter, defined by a root-mean-square value, is minimum. On the basis of such a calculation, an empirical expression for β is proposed, that takes into account the values of bending and torsion fatigue strengths at a reference number of loading cycles. According to such an expression, β is constant for a given material.

KEYWORDS. critical plane; fatigue strength; multiaxial loading.

INTRODUCTION

Structural components of machines and devices are subjected to service loads which often include multiaxial load conditions. The complex nature of the fatigue processes has produced several fatigue criteria which, implemented in algorithms, constitute a basic tool for estimating fatigue strength/life. These criteria generally reduce the spatial stress state to an equivalent uniaxial one. Among all multiaxial fatigue criteria, it is possible to distinguish a group based on the critical plane concept, which assumes that material fatigue failure is caused by stresses (strains) related to the critical plane. In 1935, Stanfield suggested the use of the critical plane to describe multiaxial fatigue [1]. Currently, such a concept gains an increasing interest.

The paper presents both a model for estimating fatigue life and the analysis of the influence of the critical plane orientation on such an estimation. Particular attention is paid to the proposal of a new function to determine the critical plane orientation, based on both the analysis of scatters and the ratio between the fatigue strength for bending and that for torsion, at the given number of loading cycles. The calculation employs the criterion of maximum normal and shear stresses acting on the critical plane [2].



FATIGUE STRENGTH EVALUATION

Generally, the estimation of fatigue strength consists of several stages. The first step includes measurement, generation or calculation of the stress tensor components according to the following equations, in the case of biaxial fatigue (for example, cyclic bending and torsion):

$$\sigma_{xx}(t) = \sigma_a \sin(\omega t) \quad (1)$$

$$\tau_{xy}(t) = \tau_a \sin(\omega t - \varphi) \quad (2)$$

where $\sigma_{xx}(t)$ refers to stress induced by bending, and $\tau_{xy}(t)$ refers to torsion-induced stress. Further:

- σ_a = amplitude of normal stress induced by bending;
- τ_a = amplitude of shear stress induced by torsion;
- ω = pulsation;
- φ = phase shift;
- t = time.

Then, the following step involves the computation of the critical plane orientation, which can be performed by using one of three established methods: weight functions, damage accumulation, variance. One damage accumulation method to determine the critical plane is that proposed by Carpinteri et al. [3], according to which the normal to the critical plane is defined by the angle β :

$$\beta = \frac{3}{2} \left[1 - \left(\frac{1}{B_2} \right)^2 \right] 45^\circ \quad (3)$$

measured with respect to the direction of the maximum normal stress, and being:

$$B_2 = \frac{\sigma_{af}}{\tau_{af}} \quad (4)$$

where σ_{af} and τ_{af} are the fatigue limits for fully-reversed bending and torsion, respectively.

As far as the multiaxial fatigue criteria based on the critical plane concept are concerned, Macha [2] formulated the criterion of maximum normal and shear stress in fracture plane for random loading, which can be generalised for different loading conditions. The general form can be written as follows:

$$\sigma_{eq}(t) = B\tau_{\eta_s}(t) + K\sigma_{\eta}(t) \quad (5)$$

where B, K are constants used for a specific criterion form [4], $\sigma_{\eta}(t)$ is the normal stress and $\tau_{\eta_s}(t)$ is the shear stress, both acting on the critical plane:

$$\sigma_{\eta}(t) = \sigma_{xx}(t) \cos^2 \alpha + \tau_{xy}(t) \sin 2\alpha \quad (6)$$

$$\tau_{\eta_s}(t) = -\frac{1}{2} \sigma_{xx}(t) \sin 2\alpha + \tau_{xy}(t) \cos 2\alpha \quad (7)$$

where



$$\alpha = \alpha_\eta + \beta \tag{8}$$

being α_η the angle defined by the direction of the maximum normal stress if the above damage accumulation method [3] is applied. An alternative method is that to determine the direction for which the normal stress variance reaches its maximum [5,6]:

$$\mu_{\eta\eta}(\alpha_\eta) = \frac{1}{T_0} \int_0^{T_0} \sigma_\eta^2(t) dt \tag{9}$$

where T_0 is the observation time interval.

The criterion proposed by Macha (see Eq. (5)) is here employed on the critical plane, where the determination of the critical plane orientation is performed according to the above damage accumulation method. The weighted factors B and K can be determined by equating Eq.(1) to σ_{af} and Eq.(2) to τ_{af} :

$$B = \frac{B_2 - \frac{\sin(90^\circ + 2\beta)}{\cos^2 \beta}}{\frac{\sin 2\beta \sin(90^\circ + 2\beta)}{2 \cos^2 \beta} + \cos(90^\circ + 2\beta)} \tag{10}$$

$$K = \frac{2 + B \sin 2\beta}{2 \cos^2 \beta} \tag{11}$$

By substituting Eq. (10) in Eq. (11), we get:

$$K = 2 - \frac{\sigma_{af}}{\tau_{af}} \tag{12}$$

According to Eq. (12), we can notice that the parameter K is a constant depending on the material fatigue properties. The final step is the calculation of the fatigue strength. For constant amplitude cyclic loading, the fatigue strength is evaluated by using Basquin's fatigue characteristics (A and m) in compliance with the relevant ASTM standard [7]. The formula for strength calculation under cyclic loading is expressed as follows:

$$N_{cal} = 10^{A - m \lg \sigma_{eq,a}} \tag{13}$$

where $\sigma_{eq,a}$ is the amplitude of the equivalent stress related to the critical plane (Eq.(5)).

MATERIALS EXAMINED

Fatigue test results related to 11 selected construction materials are analysed. According to the ASTM recommendations [7], such results are also used to calculate the regression equation for fully-reversed bending (or uniaxial push-pull):

$$\log N_f = A_\sigma + m_\sigma \log \sigma_a \tag{14}$$

and for fully-reversed torsion:



$$\log N_f = A_\tau + m_\tau \log \tau_a \tag{15}$$

being $A_\sigma, m_\sigma, A_\tau, m_\tau$ the coefficients of the regression equations for bending and torsion, respectively. The values of coefficients of the above regression equations for each analysed material are listed in Tab. 1. Eq. (3) proposed by Carpinteri et al. involves only fatigue limits, and can be applied for B_2 ranging from 1 to $\sqrt{3}$. By analysing the values of coefficients m_σ and m_τ listed in Tab. 1, all materials can be noted to be characterized by values of m_σ different from those of m_τ , which means that such materials have non-parallel mutual fatigue characteristics. Therefore, a dependence of β angle on the ratio between bending strength and torsion strength in correspondence to a given number of loading cycles, N_{fi} , is proposed in next Section.

Material	Bending		Torsion		$\beta(\Gamma_{min})$ [°]	N_{fi} [cycles]	$B'_2 = \frac{\sigma_a(N_{fi})}{\tau_a(N_{fi})}$	Number of tests
	A_σ	m_σ	A_τ	m_τ				
D30 [8]	30.50	10.75	25.40	9.20	8	2000000	1.496	6
GGG40 [9]	32.39	10.95	35.48	12.41	1	1000000	1.110	15
10HNAP [10]	30.88*	9.50*	25.28	8.20	45	2000000	1.874	108
PA4 (6082) [11]	23.80	8.00	21.40	7.70	45	2000000	1.680	45
30CrNiMo8 [12]	27.54	8.05	69.56	24.62	0	100000	1.500	9
CuZn40Pb2 [13]	19.99	5.86	45.30	17.17	16	1000000	0.920	55
GTS45 [9]	53.00	19.40	35.50	12.80	20	250000	1.265	11
Cast Iron IC2 [8]	23.7	8.80	44.00	19.50	0	1000000	1.155	4
Hard Steel 982FA [8]	36.60	12.10	49.50	18.60	14	1000000	1.550	11
SM45C [14]	31.10	10.30	49.40	18.60	0	100000	1.402	5
SUS304 [15]	19.8*	7.04*	22.5	8.7	5	2500	1.379	18

*push-pull

Table 1: Coefficients of regression Eqs. (14) and (15) and fatigue properties of the examined materials. The number of tests is also reported.

FATIGUE STRENGTH SCATTER CALCULATION

In order to analyse how the fatigue life is influenced by the value of β angle, simulation studies are carried out by assuming β ranging between 0° and 45° , with an increment equal to 1° . For each of the 46 angle values, the parameter B is computed according to Eq. (10), whereas the parameter K is a constant according to Eq. (11) and depends only on the fatigue material properties.

Fig. 1 shows the value of the parameter B against the β angle (Fig. 1(a)) and that of the parameter K (Fig. 1(b)) for 10HNAP steel [10].

In order to perform a suitable analysis of the fatigue strength scatter, the logarithmic dependence of the ratio between the experimental and calculated fatigue strength should be examined. A new method to determine such a scatter has been proposed by Walat et al. [16], who have defined the root mean square error:

$$E = \sqrt{\frac{\sum_{i=1}^n \log^2 \frac{N_{exp}}{N_{cal}}}{n}} \tag{16}$$



Therefore, the scatter can be determined as follows:

$$T = 10^E \tag{17}$$

Fig. 2 shows the relationship between the scatter value T and the angle β , for two selected materials.

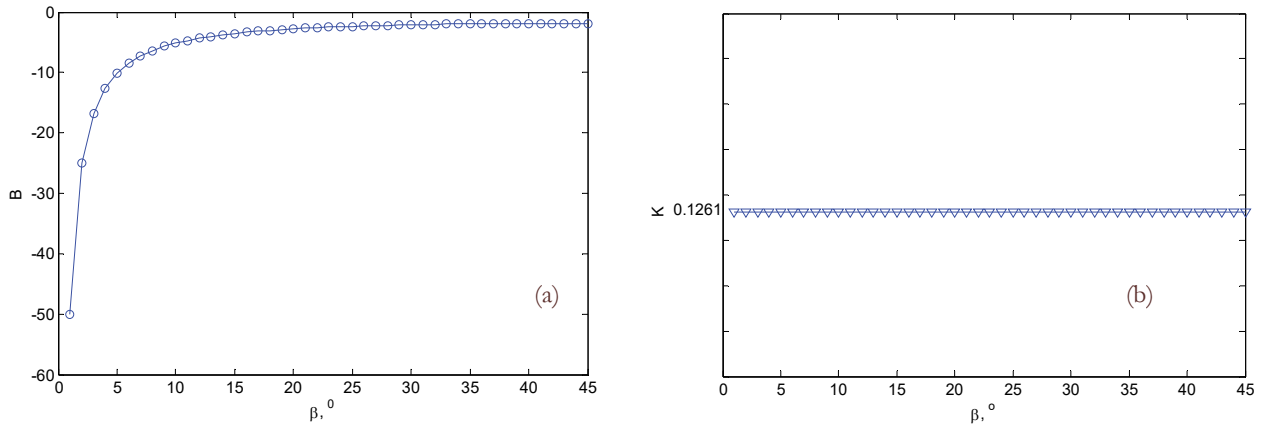


Figure 1: Dependence of the parameter B on (a) the angle β and (b) K value for 10HNAP steel.

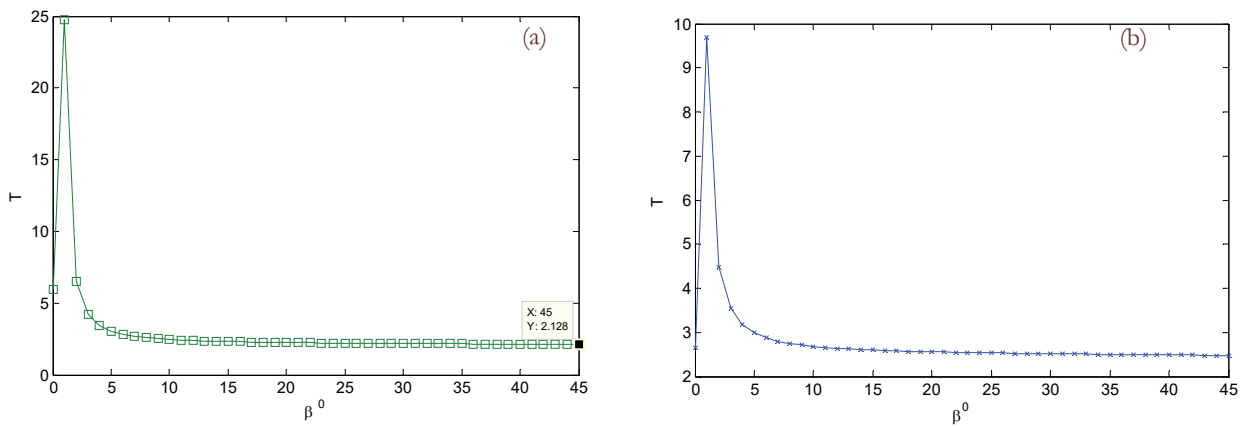


Figure 2: Relationship between the scatter T and the angle β for: (a) 10 HNAP steel [10]; (b) PA4 aluminum alloy [11].

Scatters are computed only for experimental tests under combined bending and torsion. The angle β corresponding to the minimum scatter value is registered for each examined material and listed in Table 1.

The present authors propose a new expression for β :

$$\beta = \frac{\text{arccotg}\left\{22.5 \left[\left(\frac{1 + \sqrt{3}}{2} \right) - \frac{\sigma_a(N_{fi})}{\tau_a(N_{fi})} \right] \right\}}{4} \tag{18}$$

where β is a function of the fatigue strength ratio B'_2 :

$$B'_2 = \frac{\sigma_a(N_{fi})}{\tau_a(N_{fi})} \tag{19}$$

N_{fi} is the number of loading cycles computed from Eq. (18) by inserting the value of β that minimises the scatter T . N_{fi} is considered as a material constant, and such a value is listed in Table 1 for each analysed material.

In Fig. 3, the angle β corresponding to the minimum scatter value is plotted against B'_2 for each examined material. Eq. (18) is also plotted in Fig. 3 (see the dashed curve). Note that such a relationship can be applied to a range of B'_2 larger than $\langle 1; \sqrt{3} \rangle$.

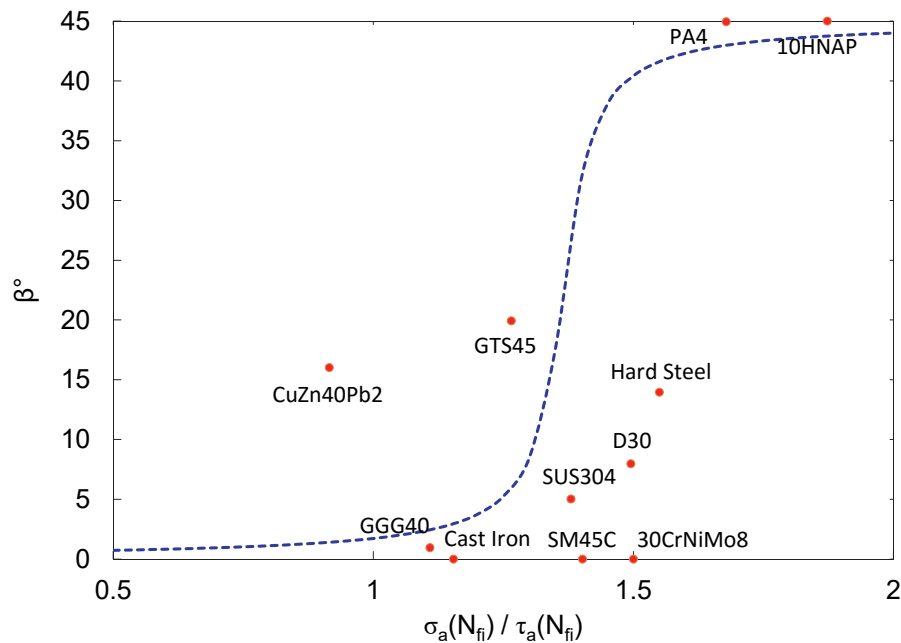


Figure 3: β against B'_2 by employing Eq. (18). The value of β in correspondence of the minimum value of the scatter is also plotted.

CONCLUSIONS

The following conclusions can be drawn:

1. In the present paper, the influence of the critical plane orientation on the fatigue strength estimation is analysed.
2. An empirical expression of the angle β used to define the critical plane orientation is proposed, the idea starting from the observation of experimental fatigue test results under combined cyclic bending and torsion.
3. Such an expression is a function of the ratio B'_2 between bending and torsion fatigue strengths at a reference number of loading cycles, and is a constant for a given material.
4. The dependence of β on the above strength ratio B'_2 (instead of the fatigue limit ratio) is here proposed for those materials characterised by m_σ different from m_τ .
5. This expression of β can be used for a B'_2 range larger than $\langle 1; \sqrt{3} \rangle$.

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