Fatigue crack growth under remote and local compression – a state-of-the-art review

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ABSTRACT. There is an ever increasing need for accurate understanding of the fatigue crack growth behaviour in major engineering materials and components. With the move towards more complex, probabilistic assessments, the traditional 'safe' or conservative approach for prediction of fatigue crack growth rate may no longer be attractive. Current codes and standards tend to be ambiguous about the treatment of compressive stress cycles: on the one hand code guidance on fatigue crack initiation may be non-conservative, while assessment of crack propagation may be inconsistently conservative. Where codes are non-conservative they could lead to dangerous assessments. The current paper provides a critical review of state-of-the-art in literature and a study of current code implications.

KEYWORDS. Fatigue crack growth; Compressive stress; Closure; R-ratio; Paris law; Design codes.

INTRODUCTION

Most common engineering predictions of fatigue crack growth (FCG) rate demand a high level of accuracy. The need for accuracy may be perceived to be of two apparently distinct natures: on the one hand the scientist and the academic is interested in deeper understanding of the fundamental behaviour of materials in order to progress what may be termed the 'frontier of science', while on the other hand the engineer is interested to utilise this, or any usable understanding in the most effective way in practical applications.

In offshore applications such as for Oil & Gas platforms, where cost saving considerations merit it, the move towards risk-based inspections and complex probabilistic assessments relies fundamentally on accurate knowledge of the fatigue crack growth behaviour. Nevertheless deterministic predictions of fatigue crack growth for safety applications have traditionally relied on inherent conservatisms in the Paris law coefficients used in the 'design' fatigue crack growth laws (fracture mechanics-based fatigue assessment), often taking the prediction curve two standard deviations away from the mean of data.

Therefore it is understandable that the apparent bias in design codes and standards, when treating the fatigue crack growth phenomenon, has been towards conservatism in the prediction of growth rates. As such, emphasis has usually not been placed on treatment of some of the beneficiary features of fatigue - such as closure or the treatment of compressive stress cycles. In some engineering components, where a crack grows in the parent material, away from any residual stresses due to welds (and the larger inherent scatter in the fatigue behaviour in this region), the mean stress could be low or even compressive (e.g. self weight) and therefore benefit may be taken from the effect of fully compressive or part-compressive stress cycles.

Although fatigue crack initiation has been observed under purely compressive remote stress cycles, current code recommendation could be misinterpreted, assuming no damage accumulation during a fully compressive stress cycle. Conversely, most national standards do not allow accounting for the beneficial effect of closure, and some even ignore the partial benefit of the compressive stress cycles in fatigue cracks growth. In BS7910 : 2013 [1], which more closely reflects UK experience and practice, the recommended FCG curves are presented for the cases of $R \geq 0.5$ and $R < 0.5$. The $R \geq 0.5$ is recommended for welded joints, and data for this family of curves is gathered from results of tests on welded samples. The data for $R < 0.5$ are gathered from a pulsating tension (minimum stress = 0 ~ 0.1) [2] taking into account the full stress intensity range [3]. No data has been used with inclusion of compressive stresses in the loading.

The ESDU Data Sheet Item 80036 [4] notes two options for calculation of ΔK when the stress cycle is partly compressive: calculating the range from the tensile part only, and calculating it from full range. The key is that the same method should be used both for predicting growth and in the presentation of the basic data. This is also in accordance with the guidance of ASTM E647 [5]. The DNV Classification Note 30.2 (1984) [6] stated that compressive stresses do not contribute to crack propagation, but the recommendation was that the whole of the stress cycle should be considered.

The current paper provides a critical state-of-the-art review of current understanding, with a view to highlighting current issues and challenges which may complicate more accurate modelling of the fatigue crack growth phenomenon in codes and standards.

Note – In keeping with the accepted nomenclature, R-ratio is defined as the ratio of the minimum to maximum stress in a give load cycle, and U is defined as the ratio of the effective stress intensity factor and the total stress intensity factor ranges.

EFFECTS OF COMPRESSIVE LOADING – TIMELINE OF OBSERVATIONS

Fatigue crack initiation and propagation under cyclic compression has been reported and investigated as early as the 60's. Almen [7] shows examples of cracks generated under compressive loading such as compressive cracking of coil springs and 'shelling' in rails. Gerber [8] and Hubbard [9] and a number of other authors observed fatigue crack initiation under compression in laboratory and components, and this is a well-documented phenomenon. See for example the works of Suresh [10] and Fleck [11] for crack initiation in steels, and Solis et al [12] for initiation in ceramics, to give a few examples.

These tests are generally conducted on notched samples, under remote compressive stress cycles. The observations from the tests were cracks that initiated at the notch, and grew to a small length and would then arrest – the final length before arrest was 0.6mm from Suresh's work [13] and between 0.68mm and 2.48mm in Fleck's tests [11].

The experiments performed by Fleck [11], Suresh [10], Pippan [12], Hermann [14] and Kasaba [15], all resulting in nucleation and growth of cracks, were all performed under fully compressive loading. In all of these experiments, the cracks grew with decreasing growth rate until complete crack arrest at a certain crack length.

Yu et al [16] demonstrated the significance of the compressive stress on fatigue crack propagation rate of aluminium alloy 2024-T351. Tests performed by Tack and Beevers [17] showed that the fatigue crack propagation rate under a negative stress ratio R is greater than that for R=0.1. Pommier [18] showed that the addition of a compressive part to the loading could increase the crack growth rate by a factor of five. The observed behaviour was attributed to plastic properties of the material and its kinematic hardening.

The general outcome of these observations is that compressive loading in notched samples can lead to fatigue crack initiation, and in the presence of purely compressive loading the crack would grow to a finite albeit small size before arrest. The fact that crack initiation and growth occurs under compressive loading is significant because in some engineering application, not foreseeing this crack could cause unexpected loss of the component. In the following sections, the factors affecting growth and subsequent arrest of these cracks are reviewed.

RESIDUAL STRESSES AND PLASTIC ZONE SIZE AHEAD OF CRACK

Many authors have associated the growth (and subsequent arrest) of compressive cracks to the residual stress field and the plastic zone generated at the notch due to the remote compressive loading, and the state-of-the art is reviewed in this section.

Saal [19] proposed a model where the residual stresses at the notch root, generated from maximum compressive load were calculated. It was assumed that under fully compressive cyclic loading, cracks would stop growing at the boundary of the plastically deformed zone. Saal's study was restricted to constant amplitude cyclic loading. It is different from Hubbard's work [9], where the residual stress field in the vicinity of the notch is not released as the crack grows. Experimental work by Saal [19] showed where no tensile stress component is present, the crack stops at the elastic-plastic boundary.

Fleck et al [11] used Dugdale's strip yield model [20] and back-calculated the effective stress intensity factor range (and hence inferred the residual stresses) from crack growth test results, using Bueckner's principle of weight function [21-22]. They found that residual stresses found by this method are greater than those calculated from Dugdale's model [11].

Jones et al [23] suggest that since negative R-ratio effects are considered to be principally caused by crack closure, a tight crack should produce more significant crack closure and hence negative R-ratio effects. However data from their tests [23] show that results from tight cracks and notches do not significantly alter negative R-ratio effects. Assuming that the reason behind this observation by is that the extent of residual stresses due to the compressive loading remains unchanged, the work of Saal [19] may explain this phenomenon. Saal shows that the condition of a specimen with a straight cut will be a reasonable approximation of calculating the plastic zone size of any elastic-perfectly plastic notched specimen. Saal's observation is further supported by Libatskii [24], who showed that the yield behaviour of static tensile tests of notched specimens of mild steel was very close to that described by Dugdale's model.

An important work was reported by Reid et al [25], which is the first work to report measured values of residual stress at a notch, generated by compressive loading, and to use this in the analysis of crack growth under cyclic compression. The measurement method has been explained in [26]. Furthermore, neutron diffraction was also used to obtain the residual stress field. It was found that the general shape of the stress distribution was independent of the pre-load used. Not surprisingly, the residual stress magnitude increased with the pre-load. The region of tensile stress near the uncracked notch tip increased in size with the value of the pre-load but in each case it was significantly smaller than the final crack length [26]. This contradicts the views of Gerber et al [8] and Suresh [10].

Reid et al [25] argue that it is possible for the crack tip to remain in a region of tension because the growth of a crack redistributes the original tensile stress to larger distances from the notch.

A number of authors have used numerical tools to model the generated residual stress field ahead of the crack, due to the compressive loading. Among them, Zhang et al [27] developed an elastic-plastic finite element model to study the compressive stress effect on fatigue crack growth under applied tension-compression loading.

Prior to this, Silva [28] had concluded that to predict fatigue crack growth behaviour under applied tension-compression loading, models based on fatigue crack closure were not suitable, and models need to be developed which are based on material's cyclic plastic properties.

The discussions above are all for fatigue crack growth. One finding worth mentioning here is by Iswanto et al [29], who found that the effect of residual stress caused by roller-working on S-N fatigue was similar to the effect of compressive mean stress. However, Chahardehi et al [30] conclude that the compressive residual stresses caused by laser peening do not influence fatigue crack growth in the way that was previously expected, i.e. superposition of stresses is not sufficient.

MODELLING THE R-RATIO EFFECT

Various authors recognized the R-ratio dependence of FCG rates, and several models have been presented to include this behaviour. Generally, equations of the Paris law type that account for the effect of R-ratio can be divided into the following three types:

1) Stress intensity factor range is defined as per the conventional definition

$$\Delta K = K_{max} - K_{min}$$

and Paris law coefficients are dependent on R-ratio. This is the approach taken in the BS7910 : 2013 recommendation [1] and ASTM E647 [5].

2) The effective stress intensity factor range ΔK_{eff} is defined as a function of ΔK and R-ratio. This is the method employed by Walker [31], and by Kurihara [32] and Eason [33], in the form of an effective coefficient U where $\Delta K_{eff} = U.\Delta K$.
3) Equations where the general form is similar to the Paris law but includes other terms such as the material toughness and K_{max}, such as Forman's equation [34], the relationship proposed by McEvily et al [35] and a number of other authors. For a review of some of the famous fatigue crack growth rate equations see the works of Bloom [36] and Huang [37]. The equations are mainly based on empirical plasticity-induced crack closure models. Additional mechanisms include asperity-induced and oxide-induced closure [36]. The Two Parameter models, e.g. as proposed by Vasudevan [38] can either be deemed to be among the second group of methods (above), or the third type of methods.

Kurihara [32] predicts that for fully reversed loading (R = -1.0), only 39 percent of the stress intensity factor range is effective for crack growth.

However, not many empirical models are based on experimental data for part compressive stresses (and fully compressive stresses, for that matter). Jones et al [23] found that by using a form of equation suggested by Eason [33], and finding the coefficients from experimental data, the effects of negative R-ratio on the fatigue crack growth rates for even the high stress range tests could be bounded by correlating the foregoing equation to only positive R-ratio test results and extending the resulting equations into negative R-ratio regime.

FACTORS AFFECTING FCG IN COMPRESSIONS - DISCUSSION

Real life applications where fatigue loading is involved present a number of complicating problems e.g. variable amplitude loading, and residual stresses, to name a couple, and some of there factor are examined in the rest of this section. The examination is by no means exhaustive, but each sub-section provides a meaningful understanding to the whole topic:

Variable amplitude loading – effect of overload and underload

Variable amplitude loading itself poses a number of complications. Variable amplitude cycles would in reality have a variable R-ratio, whereas cycle-by-cycle analysis using an R-ratio dependent growth law is tedious. Indeed, to overcome this particular issue, the loading histogram (stress range vs. number of occurrence) could be rearranged and grouped into separate subsets with similar R-ratios for ease of calculation. However, another complicating factor, which is less easily surmountable, is the influence of the previous stress cycle on the increment of growth. To demonstrate the effect of load history on fatigue, McEvily et al [39] argue that if several alloys are ranked in terms of fatigue crack growth resistance under constant amplitude loading conditions, the same order of rating may not apply under variable amplitude loading, based on findings of Minakawa et al [40]. This observation also confirms a deeper material dependence of fatigue behaviour. To provide an explanation for the effect of overload and underload (i.e. compressive overload), Silva proposed as a competition between a damage accumulation effect and a residual stress effect [41].

In a study by Stephens et al [42] it is experimentally shown that compressive loading greatly reduces the tensile overload effect. This is shown in Fig. 1 taken from Stephens [42].

McEvily et al [39] argue that "… the influence of compressive overloads or various combinations of compression-tension overloads is generally of lesser significance than a single tensile overload, an effect which can be related to a reduction of the crack opening level as the results of a compressive cycle.".

Material-dependence of the overload and underload behaviour

Silva maintains that the difference in the sensitivity of different materials to negative loads may be due to the Bauschinger effect [41]. Based on test results, Silva [43] also found that materials exhibiting strong cyclic hardening and a high Bauschinger effect were strongly affected by the compressive load while materials exhibiting no cyclic hardening were relatively insensitive to applied compressive load.

Jones et al [23] observe that experimental results found from a specific material do not necessarily generalise, i.e. do not predict the behaviour of other alloys because with the exception of corrosion-induced blunting, crack-tip blunting and crack-tip plasticity behaviour are governed by the toughness/yield ratio of the material.

In their review of fatigue crack growth, Allen et al [44] argue that where stress ratio dependence is observed, such as the Forman equation [34], it is usually associated with additional cyclic crack extension by brittle fracture or microvoid coalescence. See for example [45-46]. Where neither mechanism is operative there is generally no stress ratio

dependence. As an example, Frost et al [47] found no R-ratio dependence in pure aluminium, copper, or titanium.
In these materials, therefore, there is a mechanistic difference between higher R-ratios and cycles including negative stress, compared to materials where R-ratio dependence is observed. It may be concluded that *where empirical formulae are derived base on positive R-ratio data, extension of these formulae to negative R-ratios is not justified.*

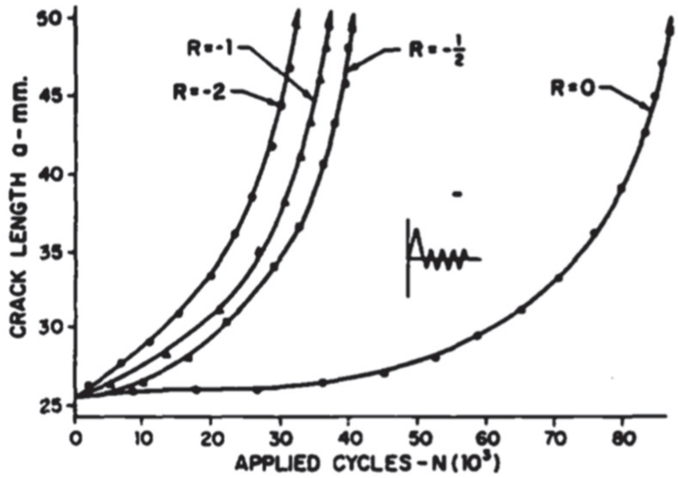

Figure 1: Crack growth at different R-ratios following single tensile overload in 2024-T3 aluminium alloy, overload ratio =2.0 [42]. Reprinted, with permission, from ASTM STP 595 Fatigue Crack Growth Under Spectrum Loads, copyright ASTM International, 100 Barr Harbor Drive, West Conshohocken, PA 19428.

A note on residual stresses

When a component contains tensile residual stresses, such as is common in weldments, the R-ratio is always assumed to be greater than 0.5. The rationale for this is that since welds can contain residual stresses of the order of the yield strength of the material, most general engineering cyclic loading cases, when superimposed on the residual stress, would result in a fully tensile cycle with high mean stresses. However, as Allen et al [48] point out, the potential for conservative prediction should be noted. Presence of tensile residual stresses implies the presence of balancing compressive stresses, which is usually not assessed in fatigue in the conservative assumption, whereas in reality in a stress-relieved weld, the crack may close during the compression cycle, therefore removing the stress concentration at the crack tip.
On the other hand, where a non-redundant structural component is welded, far-reaching residual stresses due to the misfits (or geometrical incompatibility) of the set up may be generated which have not been calculated in the modelling phase, say by using finite element method. Presence of this type of stress would increase the mean stress, and therefore should not be overlooked in assessment.

Effect of thickness

Some authors have correlated certain experimental observations to thickness effect - see for example [49]. However these explanations do not sufficiently deal with acceleration under plane strain conditions [41], and the question is still partly unanswered.

Effect of stress range

Crooker [50] showed that compressive part of the loading is more influential for low-cycle fatigue.
Jones et al [23] have examined the effect of high stress ranges in fatigue crack growth, for R=-1.0. The results suggest that there is a significant difference between fatigue crack growth rate curves for small and large stress ranges, and whether the initial stress cycle closes or opens the crack: less benefit is seen when the first cycle closes the crack. Jones's suggestion is in keeping with Crooker's observation [50].

Effect of crack length

Kurihara et al [32] performed experiments where they could show that the *U* parameter (ratio of effective stress intensity factor range and mathematical stress intensity factor range) is dependent on crack length (for smaller cracks), and this dependence is stronger for smaller (more negative) R-ratios. Fig. 2 shows this behaviour.

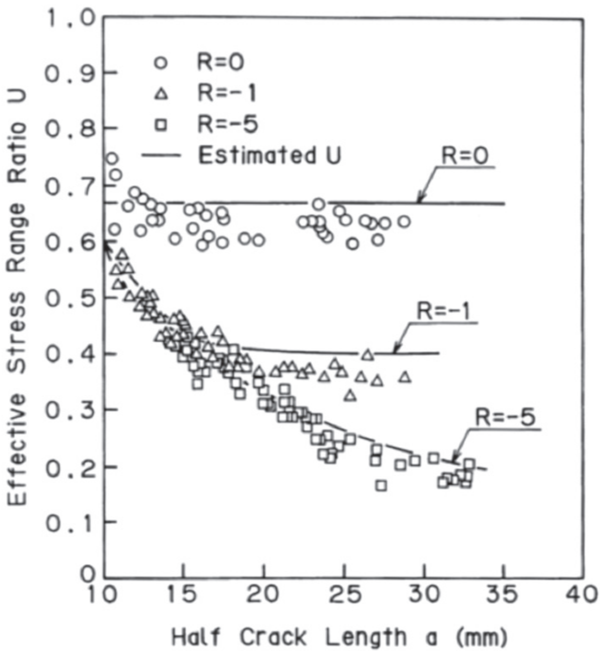

Figure 2: Dependence of *U* on crack length. Reprinted with permission from [32]. Copyright (1985) The Society of Materials Science, Japan.

During early stages of crack growth, the effective stress range ratio (*U*) decreases with an increase in crack length. This parameter then approaches a constant value. For more negative R-ratios, this decrease continues over a longer crack size. This is believed to be due to the larger size of the zone of residual stress due to compressive loading at more negative R-ratios [32].

Effect of loading mode

Only uniaxial loading is considered in the present work under mode I, i.e. 'opening mode' loading. Work on biaxial stress fields has shown similar R-ratio dependence of fatigue crack growth [51].
Chung-Seong et al [52] looked at the effect of loading mode (pure bending vs. axial loading) in closure behaviour of surface and through-thickness cracks. The loading mode does not seem to affect the relative ratio of crack opening in through-thickness and surface cracks. However, effect of loading mode on residual stresses from compressive cycle is not directly obtained. Therefore one hypothesis may be that the shape and extent of the residual stress field developed due to compressive stress could be dependent on loading mode.

Miscellaneous considerations

Care should be exercised when dealing with closure in corrosive environment, as the mechanisms involved may not have been fully understood. Also, sensitivity of closure behaviour to waveform and frequency has not been experimentally investigated, however it may be assumed that mechanistically the behaviour should not be dependent on these two parameters, as is the case in general crack growth under constant amplitude loading. Compressive behaviour at variable amplitude has also not been investigated.

Conclusions

The two important features recognized in fatigue of compressively loaded components are the beneficial effect of closure (for low R-ratio loads with a tensile part), and the finite albeit limited growth in fully compressive stress mode.
The current papers highlights the observations, performed over the last five decades, among with some of the explanations given for certain features. Fatigue crack growth in compression is clearly confirmed to be a complicated phenomenon, where a simple, broad-brush method for its assessment has not been developed. Observations include the

influences of crack length, stress range, thickness, and material properties, providing a complicated picture. However, some of the better-established trends have not been transferred to codes and standards.
A full review of the existing empirical formulae based on R-ratio, and also a full review of historic evolution of codes and standards is beyond the scope of the paper, and should be conducted in the future.

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