Multi-scale approach for the analysis of the stress fields at a contact edge in fretting fatigue conditions with a crack analogue approach

C. Montebello, S. Pommier
LMT (ENS Cachan / CNRS / Université Paris Saclay)
Claudio.Montebello@ens-cachan.fr, Sylvie.Pommier@ens-cachan.fr

K. Demmou, J. Leroux, J. Mériaux
Snecma Villaroche
Karim.Demmou@snecma.fr, Julien.Leroux@snecma.fr, Jean.Meriaux@snecma.fr

ABSTRACT. This paper describes a novel method to model the stress gradient effect in fretting-fatigue. The analysis of the mechanical fields in the proximity of the contact edges allows to extract nonlocal intensity factors that take into account the stress gradient evolution. For this purpose, the kinetic field around the contact ends is partitioned into a summation of multiple terms, each one expressed as the product between nonlocal intensity factors, Is, Ia, Ic, depending on the macroscopic loads applied to the mechanical assembly, and spatial reference fields, ds, da, dc, depending on the local geometry of the part. This description is obtained through nonintrusive post-processing of FE computation and is conceived in order to be easily implementable in the industrial context.
By using as input the macroscopic load, the procedure consists in computing a set of nonlocal stress intensity factors, which are an index of the severity of the stress field in the proximity of the contact edges.
This description has two main advantages. First, the nonlocal stress intensity factors are independent from the geometry used. Secondly, the procedure is easily applicable to industrial scale FE model..

KEYWORDS. Fretting fatigue; Contact; Fatigue.

INTRODUCTION

Computational models for fatigue damage analysis in presence of a steep stress gradient still remain poorly effective. This is the case of fretting-fatigue where a local stress concentration is introduced by contacting components experiencing small amplitude relative motion.
This phenomenon is a major concern for the aerospace industry, in particular for aircraft engine applications where there are several components that may suffer fretting-fatigue failure like the dovetail blade-root connections.
In the last years several fatigue criteria have been developed aiming at improving the accuracy of the fatigue life prediction in the presence of a severe stress gradient. Almost all of them can be divided into two major families. On the one hand, the nonlocal stress fatigue models [1, 2] employ an averaged quantity as input for the prediction criterion in order to

mitigate the effect of the strong stress gradient. On the other hand, a different approach consists in computing the evolution of the stress intensity factors, ΔK, as a function of the crack length to analyse whether the crack will stop or not [3, 4]. Even though both the approaches show good results in terms of life prediction accuracy some important limitations make them difficult to implement in the industrial context. For instance, the analysis of the ΔK evolution at the crack tip implies to set up a computational strategy to determine the crack path resulting from the macroscopic load application. This operation is heavily time-consuming. On the other hand, the methods based on an averaged quantity over a critical distance require an accurate description of the stress gradient evolution that is obtainable only by using a really fine mesh (micron size range), condition difficult to reproduce in industrial FEM models.

MODEL

Background

Fretting-fatigue is a special damage process that occurs at the contact area between two materials under load and subject to minute relative motion by vibration or some other force. One of the main peculiarities of this phenomenon is the fact that the contact introduces a severe and extremely localized stress gradient in the vicinity of the contact edge.

The feature presented above is the same that characterizes cracks or notches where a steep stress gradient is present as well. As a consequence the mechanical fields generated close to the contact edge and the ones arising at the crack tip are comparable. This analogy can be exploited to apply the mathematical tools already developed for fracture mechanics to fretting fatigue. Giannakopoulos has been the first author to quantify from an analytical point of view this analogy [5, 6]. Two different contact configuration are studied: (i) flat punch over a planar surface which creates a stress singularity at the contact tip (crack analogue) and (ii) round punch over a planar surface characterized by a smooth transition to zero pressure at the contact area edges (notch analogue).

Among the different approaches available in fracture mechanics one in particular can be really useful to describe the stress gradient effect in fretting-fatigue. An original model has been proposed by Pommier [7, 8] aiming at describing mixed-mode cyclic elastic-plastic behavior of the crack tip at the global scale. The purpose was to establish a reasonably precise model but condensed into a set of partial differential equations so as to avoid huge elastic-plastic FE computations. The model proposed hereunder exploits the results obtained by Giannakopoulos [5, 6] and Pommier [7, 8]. A description of fretting-fatigue using nonlocal quantities has been developed. The tools employed to describe the elastic-plastic behavior of the crack tip at the global scale are redefine and adapted to fretting fatigue contact problem via the crack analogue approach.

Field partitioning

The model presented here, is based on the post-treatment of the velocity field generated by fretting-fatigue close to the contact edges. The velocity field is partitioned into a summation of multiple terms, each one expressed as the product between nonlocal intensity factors, I^s, I^a, I^c, depending on the macroscopic loads applied to the mechanical assembly, and spatial reference fields, ds, da, dc, depending on the local geometry of the part,

$$\bar{v}(\bar{x},t) \cong \dot{I}^s(t)\bar{d}^s(\bar{x}) + \dot{I}^a(t)\bar{d}^a(\bar{x}) + \dot{I}^c(t)\bar{d}^c(\bar{x}) \tag{1}$$

From the practical point of view, the velocity field is obtained through a finite element computation. In Fig. 1 an example of a FE model used in the analysis, is presented. A cylinder-plain contact configuration is employed. This choice is driven by the fact that the numerical procedure is validated by comparing it with experimental tests [9-11], carried out with a cylinder-plain apparatus. In other word the FE model needs to be as close as possible to the experimental setting.

Concerning the main parameters used in the FE model definition, it consists in an elastic quasi-static computation and plane strain linear elastic quadrilateral elements are employed. To handle the contact at the interface, the technique employed is the Lagrange multipliers, which assures the best accuracy. To modelling the friction, Coulomb's friction law is employed. With regard to the mesh used, it is a structured mesh characterized by an average length of 5-10 microns. The loads applied to the pad to reproduce fretting-fatigue consist in a constant normal load (P) and a cyclic tangential load (Q). A cyclic bulk load (σ_{fa}) is applied to the specimen to reproduce the fatigue effect.

Once the velocity field is computed, the following step is to partition it in order to extract the nonlocal intensity factors. Since the most critical zone in fretting-fatigue is situated close to the contact edge, only the value of the velocity field inside a circular region Ω, of radius r, centered at the contact tip is retained.

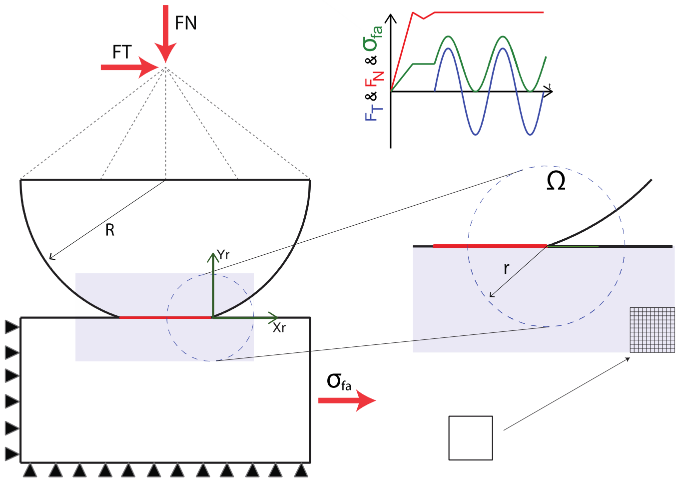

Figure 1: FE model and fretting-fatigue loading variation.

To obtain the partition presented in Eq. (1), the first step is the determination of the reference fields (da and ds). ds is selected as the field generated by a ΔP, while da as the one produced by a ΔQ in a situation where all the contact surface is in a stick condition. Further details of the procedure followed can be found in [9].
By expressing the reference fields in polar coordinates, their radial and tangential evolution with respect to the crack tip can be obtained.

$$\bar{d}^s(\bar{x}) \rightarrow \bar{d}^s(r,\vartheta) \cong f^s(r)\bar{g}^s(\vartheta) \quad (2)$$

$$\bar{d}^a(\bar{x}) \rightarrow \bar{d}^a(r,\vartheta) \cong f^a(r)\bar{g}^a(\vartheta) \quad (3)$$

To express the reference fields as a product between two functions depending separately on r and θ, the Karhunen-Loeve decomposition [10] is employed.

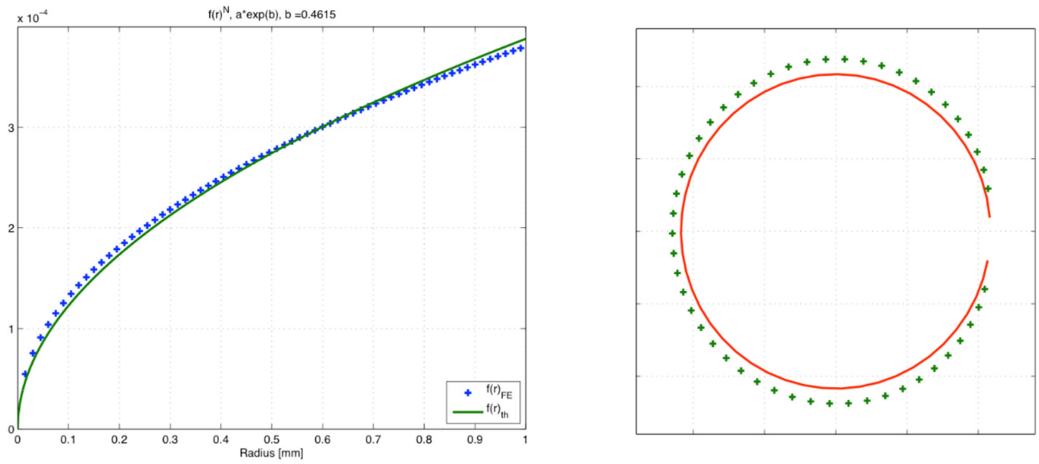

Figure 2: radial and tangential evolution of d^s.

In Fig. 2 and 3 the radial and tangential evolution of d^s and d^a are presented. Clearly the analogy between crack and contact problems is justified. Furthermore, both reference fields are normalized in order to correspond to the displacement field obtained at the crack tip during an elastic loading phase with either ΔK_I or ΔK_{II} equal to 1MPa.

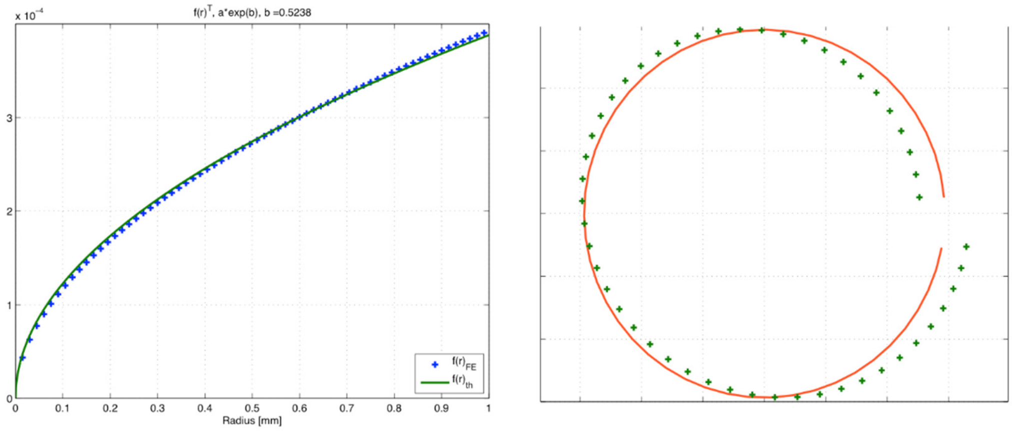

Figure 3: radial and tangential evolution of d^a.

The projection of the reference fields on the velocity field allows us to extract I^s and I^a as follows:

$$\dot{I}^s(t) = \frac{\int_\Omega \bar{v}\cdot\bar{d}^s}{\int_\Omega \bar{d}^s\cdot\bar{d}^s} \rightarrow \dot{I}_t^{\,s} = \frac{\sum_i^N \bar{v}_{i,t}\cdot\bar{d}_i^{\,s}}{\sum_i^N \bar{d}_i^{\,s}\cdot\bar{d}_i^{\,s}} \tag{4}$$

$$\dot{I}^a(t) = \frac{\int_\Omega \bar{v}\cdot\bar{d}^a}{\int_\Omega \bar{d}^a\cdot\bar{d}^a} \rightarrow \dot{I}_t^{\,a} = \frac{\sum_i^N \bar{v}_{i,t}\cdot\bar{d}_i^{\,a}}{\sum_i^N \bar{d}_i^{\,a}\cdot\bar{d}_i^{\,a}} \tag{5}$$

A first approximation of the velocity field is therefore obtained:

$$\bar{v}^e(\bar{x},t) \cong \dot{I}^s(t)\bar{d}^s(\bar{x}) + \dot{I}^a(t)\bar{d}^a(\bar{x}) \tag{6}$$

v^e is the elastic approximation of the velocity field. Since the friction between the contacting bodies introduces a nonlinearity in the system a third term, v^c, has to be added to the approximation presented in Eq. (6),

$$\bar{v}^c(\bar{x},t) = \bar{v}(\bar{x},t) - \bar{v}^e(\bar{x},t) \tag{7}$$

Here, the Karhumen-Loeve decomposition is used twice: the first time to partition the residual velocity field in a product between two functions depending separately on time and space,

$$\bar{v}^c(\bar{x},t) \cong \dot{I}^c(t)\bar{d}^c(\bar{x}) \cong \dot{I}^c(t)\, f^c(r)\,\bar{g}^c(\vartheta) \tag{7}$$

and the second time to analyze the evolution of dc with respect to r and θ, as presented in Fig. 4. Also in this case the field is normalized in order to assure that fc(r=0) is equal to 1. By contrast with the elastic reference fields, here the intensity decreases quickly far from the contact confirming the extremely localized effect of friction.
The error introduced by the approximation can be defined as follows:

$$\xi_e(t) = \sqrt{\frac{\int_\Omega \left[\bar{v}(\bar{x},t) - \bar{v}^e(\bar{x},t)\right]^2}{\int_\Omega \bar{v}(\bar{x},t)^2}} \tag{8}$$

$$\xi_t(t) = \sqrt{\frac{\int_\Omega \left[\bar{v}(\bar{x},t) - \bar{v}^e(\bar{x},t) - \bar{v}^c(\bar{x},t)\right]^2}{\int_\Omega \bar{v}(\bar{x},t)^2}} \tag{9}$$

where ξ_e is the error introduced considering just the elastic approximation while in ξ_t all the terms presented in Eq. (1) are taken into account. In Fig. 5 the evolution of the error during a fretting-simulation is shown. It is worth noting that the total error falls below 5 %.

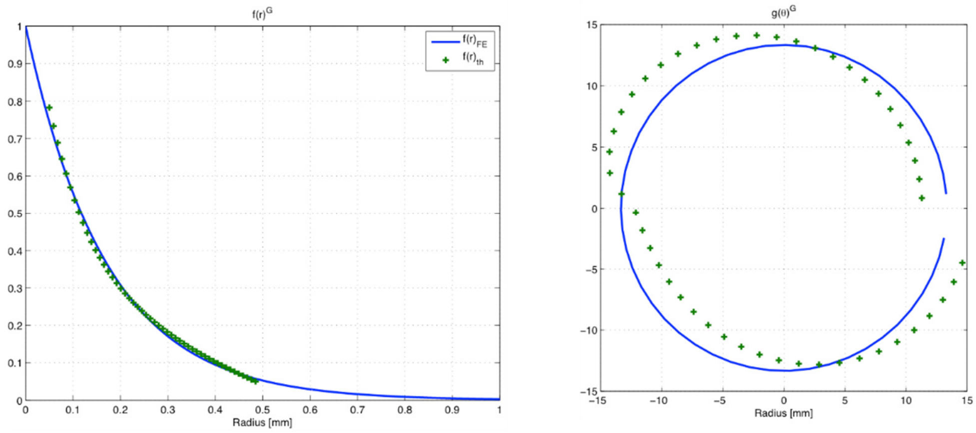

Figure 4: radial and tangential evolution of dc.

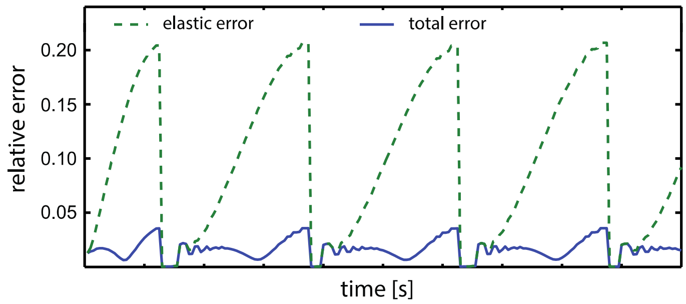

Figure 5: Error evolution during fretting-fatigue simulation.

The maximum of the cumulative integral of the intensity factors rate is computed in order to obtain, I^a, I^s, I^c.
To resume for any given macroscopic load combination (P, Q, σ_{fa}), a univocal set of nonlocal intensity factors (I^a, I^s, I^c) is found which characterizes completely the stress field close to the contact edge.

EXPERIMENTAL VALIDATION

The procedure described in the previous section has been validated by comparing it to the experimental results coming from the work of Fouvry [2]. The author performs several plain fretting tests using a cylinder-plain apparatus in order to determine different crack initiation frontiers as a function of the pad radius used. The material employed in the experimental campaign is 35 Ni Cr Mo 16 low-alloyed steel with the following properties:

- Young's modulus = 205000 MPa
- Poisson's ratio = 0.3
- Yeld stress (0.2%) = 950 MPa
- Ultimate stress= 1130 MPa
- Traction compression fatigue limit (R=-1, 10^7 cycles) = 575 MPa
- Shear fatigue limit (R=-1, 107 cycles) = 386 MPa
- Long crack threshold, ΔK_0 = 3.2

Concerning the test apparatus, the plain fretting tests are carried out using a tension-compression hydraulic system as presented in Fig. 6. The normal force (P) is kept constant, while a purely alternating cyclic displacement (δ) is imposed. As a consequence, a cyclic tangential load (Q) is generated at the contact surface. After a certain number of cycles, usually 10^5 or 10^6, the test is stopped and the specimen is cut along the median axis of the fretting scar in order to verify whether the crack is initiated or not. This procedure is repeated several times to find the tangential load amplitudes (Q*) that provokes crack initiation for a given normal load. Here, the crack is considered "initiated" if its projected length is higher than 10 μm.

To highlight the stress gradient effect, this process is reiterated for different cylinder radii. Varying the normal force, the normal pressure (p_0) at the contact surface is kept constant and, therefore, the eventual difference in the initiation threshold needs to be caused by different stress gradients generated by the change in geometry.

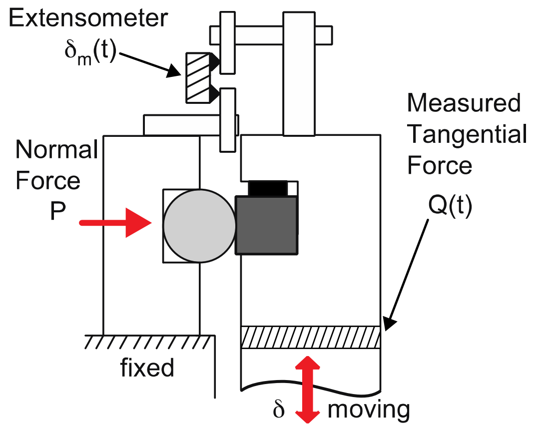

Figure 6: schematic representation of a plain fretting apparatus.

In Fig. 7, the experimental crack initiation frontiers are presented. It is clear that the gradient effect plays an important role. In particular it is evident that the biggest is the pad radius the worst is the effect in term of crack initiation threshold. To interpret this tendency, it is important to notice that two factors influence crack nucleation; (i) the maximum stress at the hot spot and (ii) the material process volume over which the maximum stress operates. The increase in pad dimension extends the influence of contact stress below the surface, and therefore increases the process volume.

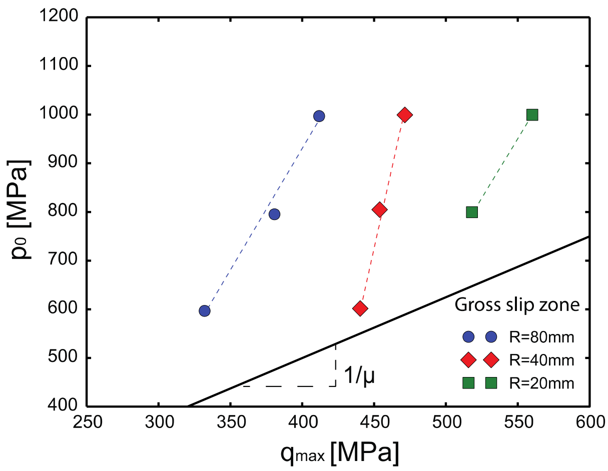

Figure 7: Crack initiation frontier for 35NiCrMo16 low-alloyed steel.

Application of the methodology.

The steps followed in the application of the numerical algorithm presented above are the followings (Fig. 8):

1. The experimental data highlight different crack initiation frontiers expressed in (q_{max}-p_0) coordinates. For each point belonging to these frontiers the macroscopic load to apply in the FE model are computed. For cylinder-plain contact configuration, analytical expressions linking q_{max}-p_0 to P and Q are available.
2. A FE computation is run and the velocity field is computed. The forces used as input to load the structure are the ones found in the previous step.
3. The velocity field is partitioned as presented in Eq. (1).
4. The nonlocal stress intensity factors are extracted and a nonlocal crack initiation map is defined.

Before to comment the results shown in Fig. 8, it is worth noting that only the elastic nonlocal stress intensity factors (I^a-I^s) are used in the description of the crack initiation frontiers for two reasons. The third term, I^c, describe the nonlinear effect of friction, which is extremely localized as confirmed by Fig. 4. As a consequence the elastic nonlocal intensity

factors, I^s and I^a, are sufficient to univocally characterize the contact edge stress condition. This approach is analogue to the one followed in fracture mechanics where the hypothesis of small scale yielding permits to use an elastic stress intensity factor, K, to describe the mechanical field close to the crack tip. On the other hand, all the test data are obtained assuring that the contact is in a partial slip regime. As explained by Vinsdbo and Sodemberg in [11], in this condition the wear rate due to friction is negligible.
The outcome of the application of the numerical algorithm to extract nonlocal intensity factors (Fig. 8), shows clearly how these quantities are objective quantities through which is possible to take into account the gradient effect efficiently. The different crack initiation frontiers displayed in a classical q_{max}-p_0 crack initiation map, merge into a single one if the nonlocal intensity factors are used.

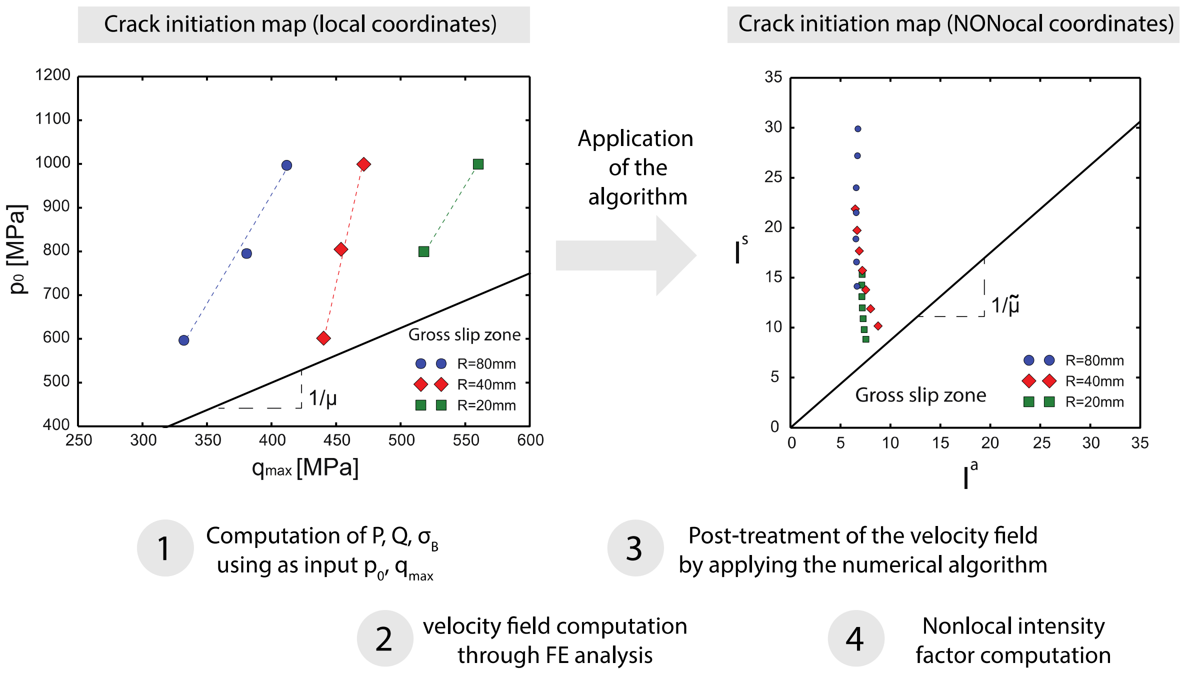

Figure 8: different steps in the application of the numerical algorithm.

Conclusions and Prospects

In order to apply the methodology described above, it is useful to remember the main challenges that the industrial sector has to handle when the structures are interested by fretting fatigue problems.
The first main problem is related to the fact that fretting-fatigue introduces a severe stress gradient at the contact interface that depends on the local geometry of the part. The fatigue life of the part depends on the stress gradient, which in turn depends on the local geometry. As a consequence, if the example of an aircraft engine is considered, the manufacturer has to certify experimentally several parts to show that no catastrophic failures occur. The limitation here is that the certification performed for a given part cannot be used for another part sharing the same properties except for the geometry. This is a big limitation because it forces the manufacturer to multiply the experimental tests to certify all the different components.
With the methodology introduced here, we propose a possible solution to this problem. If the nonlocal stress intensity factors are used to describe the mechanical field arising close to the contact edges, the gradient effect is taken into account and the change in geometry is no more a problem.
A second important complication is due to the fact that the few multi-axial fatigue criteria suited to be used for fretting fatigue [1- 4], need a really precise description of the stress gradient evolution. The only way to obtain it is by using FE computations characterized by extremely fine meshes. This requirement is not compatible with industrial constraints.
The methodology presented here is well-suited for a multi-scale approach. The nonlocal stress intensity factors can be extracted from rough meshes. In addition, since the procedure is not intrusive, it can be easily implemented in industrial procedures.

The future works will focus on the development of a fatigue criterion that take as input the nonlocal stress intensity factors extracted from the post-treatment of the velocity field. In this way it should be possible to place the position of the crack initiation frontier easily.
The second main objective is to apply the procedure to an industrial part. In particular the disc-blade root attachment will be analyzed.
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