



## Can $\Delta K_{eff}$ be assumed as the driving force for fatigue crack growth?

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**ABSTRACT.** This work raises some new questions about the validity of blindly assuming that Elber's effective stress intensity factor is the actual fatigue crack driving force, and that as so it can be used to explain all load sequence effects on fatigue crack growth (FCG). Although plasticity-induced crack closure can be a quite reasonable heuristic explanation for many non-elementary FCG behaviors, it has some limitations that cannot be ignored. In fact, this never settled discussion is particularly important for the simulation of FCG lives under real service loads, a most important practical issue. After arguing that  $\Delta K_{eff}$  can spoil the use of the most important similitude principle in FCG problems, simple but convincing experimental data that cannot be explained by this classical idea is presented here. This data involves the shape of fatigue crack fronts and the FCG behavior under nominally plane stress and plane strain conditions.

**KEYWORDS.** Sequence effects on FCG; Near and Far Field Opening Loads; Crack Front Shapes.

### INTRODUCTION

Fatigue crack growth rates are very much susceptible to brusque changes in crack driving forces, which may cause important load sequence effects by significantly altering subsequent rates as compared to the rates induced by identical driving forces that have not been previously affected by sudden load changes. Such effects include delays, arrest, or even acceleration of FCG rates after tensile overloads (OL) or abrupt decreases in the applied stress intensity factor (SIF) range  $\Delta K$  and/or peak  $K_{max}$ ; sudden fracture caused by very large OLs; and reduction of OL-induced delays after compressive underloads (UL). The order of variable amplitude load (VAL) events can thus have a huge influence on FCG lives, particularly in cracked components that must tolerate rare but significant OLs during their duties. In fact, FCG lives simply cannot be properly estimated in many if not most practical applications if such *load history effects* are neglected. Such effects can be induced by several mechanisms that can be divided into three main classes [1]: (i) fatigue crack closure induced by plasticity, roughness, phase transformation, and/or oxidation, all mechanisms that act on the crack faces, thus *before* the crack tip; (ii) blunting, kinking, or bifurcation of the crack tip, mechanisms that act *at* the crack tip; and (iii) residual stresses and/or strains, mechanisms that act *ahead* of the crack tip. Moreover, the importance of the various mechanisms that can induce load order effects on FCG may depend on many factors, like for example: the sizes of the crack and of the residual ligament  $r_l$ ; the transversal constraints around the crack tip; the residual stress state around the crack tip; the load and the overload (OL) ranges and maxima; the microstructure of the material; the number of OL cycles; and the environment.

In many practical cases one of those mechanisms can be so dominant that the others may become negligible, but in other cases they may be not. Such mechanisms can act competitively reducing the effects of the others (e.g. crack tip bifurcation after an OL can reduce its opening load and decrease the subsequent influence of crack closure), or else, they can act symbiotically (e.g. as martensite is less dense than ferrite, martensitic transformation induced by plasticity increases the material volume inside the plastic zones, thus tends to increase residual stresses ahead, as well as crack opening loads

behind the crack tip). Since too many variables can affect the FCG behavior under VAL, it is no surprise to still find controversy in the vast literature about this important subject. Many experts vigorously defend that Elber's crack closure is the single or at least the dominant cause for all load order effects, whereas others simply deny that plasticity-induced crack closure may have any importance on FCG. Moreover, there are respectable arguments and even sound experimental data to justify such different points of view [2-6], sometimes sustained in a regrettably radical way.

Anyway, it is an undisputable fact that cracks do not grow by fatigue through virgin material. Instead, they propagate by small growth steps that cut material previously deformed by the monotonic and reverse plastic zones  $p_{\zeta}^{\zeta}$  and  $p_{\zeta}^{\zeta}$  that always follow their tips, leaving an envelope of residual strains around their faces. Moreover, as their uncracked residual ligaments normally remain elastic during most of their lives, fatigue cracks usually grow accompanied by  $p_{\zeta}^{\zeta} < p_{\zeta}^{\zeta} \ll r_l$ . According to Elber, plasticity-induced crack closure is caused by such elastic  $r_l$ , which tend to compress the plastified envelopes that wrap fatigue cracks as they try to recover their previous shape when unloaded. So, when a crack closed by its  $r_l$  is reloaded, it should first gradually relieve the compressive loads transmitted through its faces until it reaches its opening load  $K_{op} > 0$ . This behavior can be macroscopically detected by compliance measurements [7-9], see Fig. 1, although some modern 3D microtomography studies [10-11] question the existence of a clearly defined opening load.

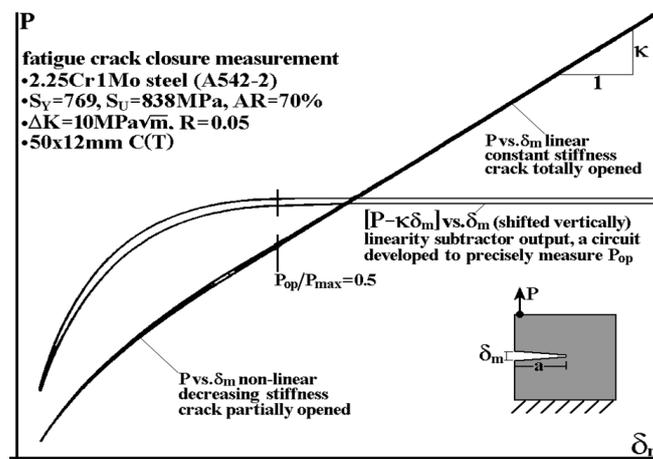


Figure 1: Macroscopic fatigue crack opening load measurement from compliance plots that display the load  $P$  versus the load point displacement  $\delta$  (or vs. a suitable strain  $\varepsilon$ , e.g. the back face strain, which is proportional to  $\delta$  in linear elastic components). Note that  $[P(\delta_m) - \kappa\delta_m] \times \delta_m$  is the signal from the linearity subtractor, an interesting equipment described in [9], and that  $\delta_m$  is the crack mouth displacement, which is also proportional to  $\delta_p$ , the load point displacement.

Using a highly simplified 2D view of the FCG problem and arguing that fatigue crack tips cannot be reloaded before they are completely opened, so that they cannot grow when still closed under loads  $K < K_{op}$ , Elber defined an effective stress intensity factor (SIF) range that would be responsible for FCG:

$$\Delta K_{eff} = K_{max} - \max(K_{op}, K_{min}) \leq \Delta K = K_{max} - K_{min} \quad (1)$$

Elber also assumed that  $\Delta K_{eff}$  (instead of  $\Delta K$ ) controls FCG rates. If it really does so, then any load event that alters  $K_{op}$  also affects subsequent FCG rates. In such cases, OL-induced FCG rate delays would be due to the larger reaction of elastic residual ligaments over the plastic zones hypertrophied by the overloads,  $p_{\zeta OL}$ , when compared to the  $r_l$  reaction over the smaller  $p_{\zeta}$  that accompanies crack tips under normal loads (not affected by their previous histories). So, using simplified macroscopic 2D arguments,  $K_{op}$  should increase when the crack penetrates  $p_{\zeta OL}$ , decreasing  $\Delta K_{eff}$  and delaying subsequent FCG rates. This plausible hypothesis can reasonably explain why FCG rates decrease after OLs, at least in plane stress ( $p$ - $\sigma$ ) cases. It can also explain the existence of FCG thresholds and why ULs can decrease the beneficial effects induced by OLs (they tend to reduce the tensile residual strains inside  $p_{\zeta OL}$ , hence to decrease  $K_{op}$  and to increase  $\Delta K_{eff}$ ).

Elber's model is popular because it seems reasonable to assume that  $K_{op}$  should increase after the crack tip penetrates  $p_{\zeta OL}$ , decreasing  $\Delta K_{eff}$  and delaying the crack. In this simplified 2D macroscopic point of view, the OL should first locally blunt the crack tip, decreasing  $K_{op}$  and accelerating the crack before it penetrates  $p_{\zeta OL}$ . The maximum retardation should occur after the crack grows for a while after the OL, a phenomenon called delayed retardation. Moreover, as the residual strains inside  $p_{\zeta OL}$  decrease from the OL application point up to its boundary, OL-induced delay effects should decrease while the crack gradually crosses  $p_{\zeta OL}$ , and end after the plastic zone that follows the crack tip leaves it (assuming FCG under



otherwise fixed load conditions). This idealized phenomenology was experimentally verified in thin plates, soon after the plasticity-induced crack closure concept was proposed, another reason for its quick acceptance [12]. The same idea can be applied to justify load order effects caused by abrupt decreases in  $\Delta K$  and/or  $K_{max}$ . Many articles support such hypotheses [13-15], but most of them deal with delay effects measured on phase II FCG under relatively high  $\Delta K$  and low  $R = K_{min}/K_{max}$  in dominant plane stress conditions, with  $p\zeta$  sizes not much smaller than the specimen thickness. The  $R$  effect on  $K_{op}$  has been studied e.g. by Newman, Schijve, and Topper, see Fig. 2 [16-18]. According to the predictions shown in this figure, high- $R$  FCG should be closure-free especially under plane strain ( $p\ell$ - $\epsilon$ ), as discussed in [1].

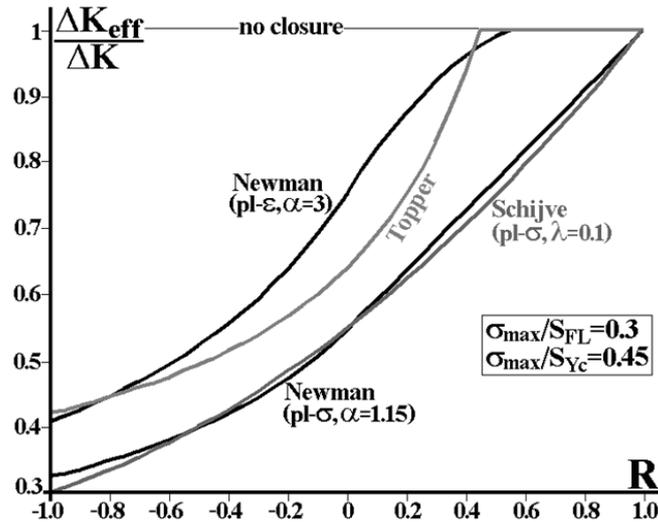


Figure 2:  $\Delta K_{eff}/\Delta K$  versus  $R$  predictions by Newman, Schijve, and Topper models ( $\lambda$  is a parameter in Schijve's model). So, Neither Newman's model predicts crack closure under  $p\ell$ - $\epsilon$  for  $R > \sim 0.5$ , nor does Topper's model for  $R > \sim 0.4$ , for this  $\sigma_{max}$  level [1].

### ISSUES WITH $\Delta K_{EFF}$ AS THE FCG DRIVING FORCE

If classical closure predictions like the ones mentioned above are true, and if  $\Delta K_{eff}$  indeed controls FCG rates, then the basic Fracture Mechanics' similarity principle based on SIFs could be questioned. Indeed, whereas the SIFs of cracked components can be listed, their crack opening loads  $K_{op}$  cannot, because they are not unique for a loading/geometry pair. In fact,  $\Delta K_{eff}$  values depend on the applied stress levels and at least on the cracked piece thickness  $t$  as well. Moreover,  $K_{op}$  may depend on the residual ligament size too, and there is no general model that can account for all such factors in generic structures yet. So, if  $\Delta K_{eff}$  controls FCG, based on the analyses studied in the previous sections the fatigue lives of relatively thin pieces under  $p\ell$ - $\sigma$  FCG (with a large  $p\zeta/t$  ratio) should thus be larger than the lives of similar but thicker pieces that work under equally fixed loading conditions ( $\Delta K$ ,  $R$ ) in  $p\ell$ - $\epsilon$ . Besides, even under such simple conditions, if the crack starts to grow under  $p\ell$ - $\epsilon$ , as they usually do when they are small, and gradually changes to a  $p\ell$ - $\sigma$  dominated stress state as its size increases, then FCG rates should vary in the same piece between these two limit cases as the crack sizes increases. Hence, not even  $da/dN \times \Delta K_{eff}$  data would provide enough information on the FCG behavior of structural materials. So, without the  $K$ -similarity, it would be very difficult to reliably predict residual lives of cracked structures even in very simple practical applications.

However, unlike in fracture predictions, thickness effects usually are not a major concern for FCG applications, and  $da/dN \times \Delta K$  (instead of  $da/dN \times \Delta K_{eff}$ ) curves keep on being reliably used for residual life predictions in most structural integrity analyses. Fig. 3 shows some data that supports this practice: its points are much less scattered if plotted against  $\Delta K$  than  $\Delta K_{eff}$  [19]. Due to the low  $R = 0.05$  value, crack closure was clearly identified during such tests, and the opening load  $K_{op}$  needed to calculate the effective SIF range  $\Delta K_{eff}$  in this figure was properly measured using strain gages bonded on the back face of DC(T) specimens and the linearity subtractor technique described in [9]. Such data also supports ASTM E647 standard procedures, which suggest but do not impose thickness limits for the specimens it accepts to measure  $da/dN \times \Delta K$  curves, implicitly accepting that  $\Delta K$  (instead of  $\Delta K_{eff}$ ) is the parameter that controls FCG under fixed  $R$ .

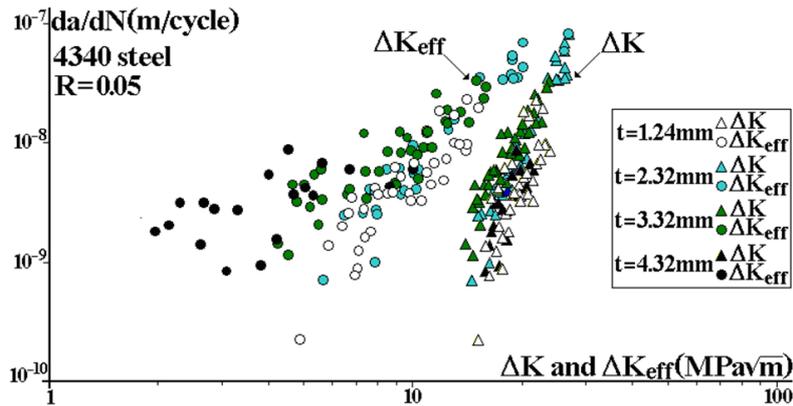


Figure 3: FCG rates plotted as a function of  $\Delta K$  and of the measured effective range  $\Delta K_{eff}$ .

Although not a hard evidence against the idea that  $\Delta K_{eff}$  is the FCG driving force, another test clearly demonstrates that a partially closed fatigue crack can grow in the portion of its front that opens under tensile loads, while the portion that is under compression stands still [21]. In this way, it also shows that FCG driving force gradients along the crack fronts can induce different growth rates along them, so that they can distort the crack front shape as the crack grows.

First, a mode I edge crack was grown for a while in SE(T) plate-like specimens under pure tension loads at  $R \cong 0.05$ , generating approximately straight fronts as usual. Then the pre-cracked specimens were repositioned and reloaded under pulsating 4-point bending, working in this way as a beam with a side crack, see Fig. 4. The main objective of such tests was to verify 3D modeling procedures needed to describe 2D FCG considering contact stresses along a portion of the crack face, an interesting non-trivial numerical modeling problem, see [21] for details. However, since there is no doubt that the bent cracks partially close their fronts due to the compressive stresses induced by the bending loads, they can verify as well how such cracks propagate under variable driving forces along their fronts. Moreover, since the partial closure of their fronts is induced by the variable bending loads along it, it is independent of Elber's or of any other type of closure mechanism. Thus, their behavior does not depend on any assumption about their actual driving forces, i.e., it does not matter if they are driven by  $\{\Delta K, K_{max}\}$  or by  $\Delta K_{eff}$ .

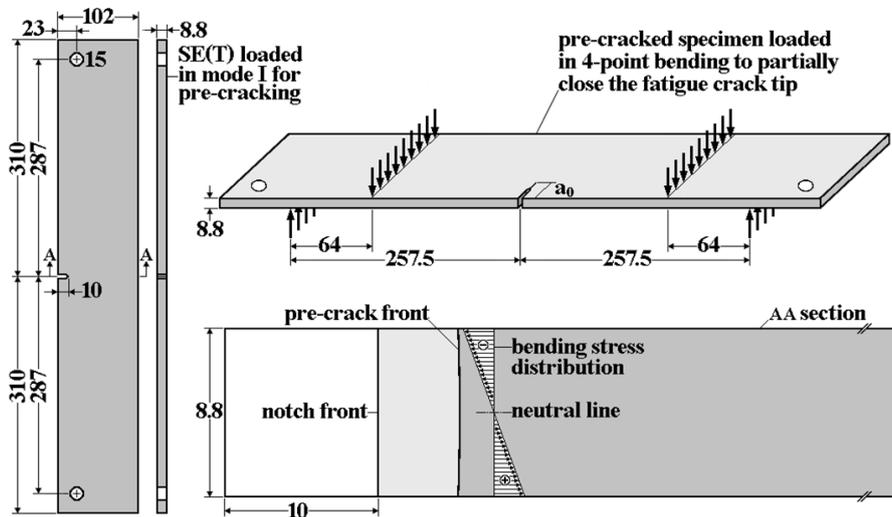


Figure 4: Pre-cracked SE(T) specimen loaded in pure bending to partially close its crack.

Anyway, the cracks tested under transverse bending severely distorted their initially (quasi) straight fronts as they grew after the load applied on the pre-cracked plates changed from pure tensile to pure bending, see Fig. 5. Note that the successive crack fronts depicted in this figure clearly show that although the pre-crack front started almost linear, it slowly assumed an increasingly pronounced L-shape after partially closed during its subsequent 2D FCG under pulsating bending loads. This is exactly the expected behavior of a crack front that advances by fatigue depending on the *local* value of the crack driving force along it.

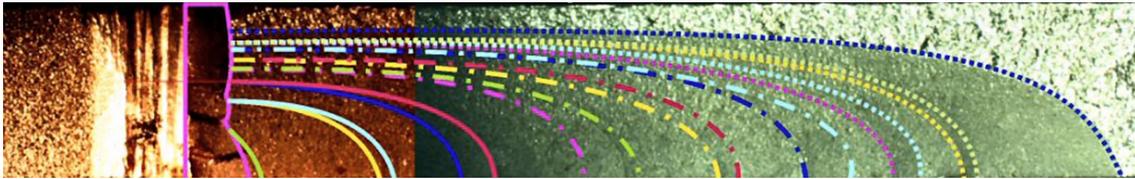


Figure 5: Successive crack fronts propagated in bending from an initially straight shape.

In other words, such tests confirm the (reasonable) idea that FCG is a local phenomenon, i.e., that it is induced by the local value of the driving force along the crack front. Consequently, the characteristic approximately parallel striations generated by the homologous propagation of crack fronts at every load cycle that so nicely illustrate the gradual nature of the FCG process in most fatigue textbooks would require an iso-driving force along them, see [22] for a sound *mechanical* evidence that supports this claim. Consequently, if the fatigue cracks are subjected to driving force gradients along their fronts (like e.g. after tensile OLs pin the  $pI-\sigma$  part of the crack front, as McEvily claims is the mechanism responsible for FCG delays in his nice experiments [23-24]), then the subsequent crack fronts should tunnel or bow with increasing curvature while the crack is delayed by the OL. If the crack fronts do not tunnel, then their driving force should be constant along their fronts, with no difference between  $pI-\sigma$  and  $pI-\varepsilon$  regions, caused by plasticity-induced closure or by any other mechanism. It is quite surprising that this powerful argument is not widely used in fractographic studies of fatigue surfaces, particularly those obtained under variable amplitude load tests.

A third data set reinforces the idea that although Elber's plasticity-induced closure certainly is a plausible mechanism to explain many peculiarities of the FCG behavior, it has at least some limitations. In these simple but discriminating fatigue tests the cracks were grown under quasi-constant  $\Delta K$  and  $R$  loading conditions in thin and thick specimens of the same material, carefully measuring both the FCG rate  $da/dN$  and the crack opening load  $P_{op}$  while the cracks grew. The test specimens thickness  $t$  was chosen to guarantee  $pI-\sigma$  conditions (making  $p\bar{z}/t \cong 1$ ) in the thin specimens and  $pI-\varepsilon$  conditions in the thicker ones, assuming that classical ASTM E399  $pI-\varepsilon$  requirements can be used in fatigue as well (making  $t > 2.5(K_{max}/S_Y)^2$ ). All the test specimens were cut from an as received 1020 steel 3" wrought round bar with yield and ultimate strengths  $S_Y = 262\text{MPa}$  and  $S_U = 457\text{MPa}$ . The applied load was maintained approximately constant by adjusting the load at small crack increments, following traditional standard ASTM procedures [20]. The crack length was redundantly measured by compliance and by optical methods, using a strain gage bonded on the back face of the specimens and a traveling microscope. The crack opening load  $P_{op}$  was also *redundantly* measured by the procedures studied in [9], using the back face strain gage and a gage strip with several strain gages bonded ahead of the notch tip, see Fig. 6. The  $P_{op}$  values measured from the various gages showed no discrepancy, meaning the same value was obtained from the near and from the far-field strain signals. The crack face of one of the thick specimens with its homologous crack fronts is shown in Fig. 7.

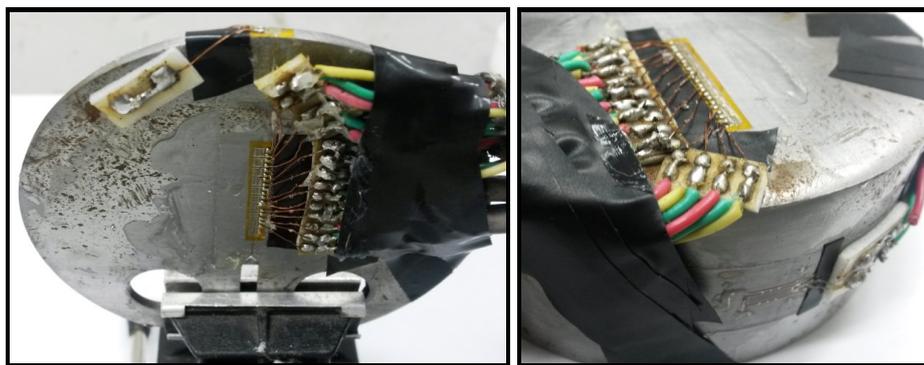


Figure 6: Gage strips and back face strain gages bonded on a thin and on a thick DC(I) TS.

Standard 76mm diameter DC(I) specimens were cut perpendicularly to the round 1020 steel bar axis, but with two very different thicknesses. The thinner specimens were loaded under  $\Delta K = 20\text{MPa}\sqrt{m}$  and  $R = 0.1$  and, to force the crack to grow under nominally  $pI-\sigma$  conditions, they had  $t = 2\text{mm} < p\bar{z}_{max} = (1/\pi) \cdot (K_{max}/S_Y)^2 = (1/\pi)[20/(0.9 \cdot 262)]^2 = 2.29\text{mm}$  (using Irwin's estimate for the maximum plastic zone ahead of the crack tip, assuming that this traditional 2D view is appropriate

to define a plane stress state in FCG). The thicker ones were loaded under  $\Delta K = 20 \text{MPa}\sqrt{\text{m}}$  and  $R = 0.05$ , and to force the crack to grow under predominantly  $pI\text{-}\varepsilon$  conditions they had  $t = 30 \text{mm} > 2.5 \cdot (K_{\text{max}}/S_Y)^2 = 2.5 \cdot [20 / (0.95 \cdot 240)]^2 = 16.14 \text{mm}$ , using the ASTM E399 criterion to define a plane strain state in FCG, as mentioned before.

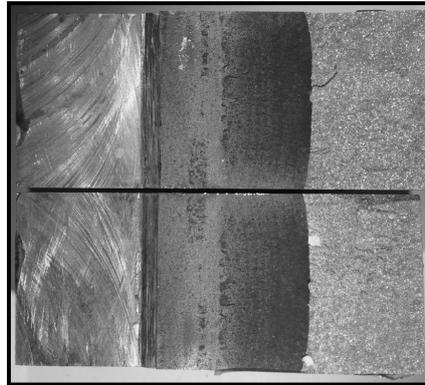


Figure 7: Cracked surface of one thick specimen, showing its homologous successive crack fronts, an evidence that it grew under an iso-driving force across the TS thickness.

Four such DC(T) specimens were fatigue tested, two thin and two thick ones. It must be emphasized that the loading conditions were maintained essentially constant, and that no overloads or similar events that could possibly alter the crack propagation behavior were applied on the tested specimens. The  $da/dN$  FCG rates and the crack opening ratios  $K_{op}/K_{\text{max}}$  measured along the crack propagation path are shown in Fig. 8-9. Note that in those figures the crack size is quantified by  $a/w$ , the ratio between the crack length  $a$  and the uncracked ligament size  $w$ , measured from the load line. Those are the simplest tests that can possibly be made to verify the basic question “is  $\Delta K_{\text{eff}}$  the actual driving force for FCG?” Indeed, the only fancier detail in such tests was the careful and continuous measurement of the opening loads while the cracks propagate, made with the help of a specially developed software written in LabView to numerically implement the straight line fitting and the linearity subtractor techniques to the redundantly measured  $P \times \varepsilon$  near and far field signals [9]. The FCG experiments were performed in an Instron 100kN servohydraulic testing machine and the data acquisition was made using National Instruments NI 9215, NI 9235, and cDAQ-9172 instruments.

It must be emphasized as well that the opening loads macroscopically measured from compliance curves as originally proposed by Elber were obtained using the best available techniques, as discussed elsewhere [7-9]. Such measurements are the best way to identify if  $\Delta K_{\text{eff}}$  is indeed the FCG driving force in any given fatigue experiment, since it eliminates any doubt about the opening load, at least from the macroscopic point of view used to defend (or to deny) Elber’s ideas [13]. Moreover, since in relatively recent works some important groups are claiming that such closure measurements should be performed on  $P \times \varepsilon$  curves measured using the near field strains close to the crack tip instead of the far field strains measured e.g. in the back face of the test specimen [25], in the tests presented here *both* measurements were redundantly made. However, no difference was observed in the opening loads identified by both techniques. In fact, even if such signals lead to different  $K_{op}$  values, the approximately homologous crack fronts would indicate that they propagated under macroscopically uniform driving forces.

Note in Fig. 8-9 that the measured FCG rates in the four tested specimens can be clearly bounded by a single dispersion line, so it can be said they were essentially equal independently of the TS thicknesses. This simple and reliable experimental result certainly confirms the traditional ASTM view that  $da/dN \times \Delta K$  rates measured under fixed R-ratios can properly characterize the FCG of structural materials, at least when applied to the tested steel. This result also confirms the more physically appropriate idea that  $\{\Delta K, K_{\text{max}}\}$  can be viewed as the FCG driving forces, so it reassures that  $\Delta K$  can be indeed used as a similitude parameter in FCG predictions.

However, this data can be used as well to question the alternative view that FCG is driven by  $\Delta K_{\text{eff}}$ . Since those tests follow straightforward procedures, and since they clearly show that the crack opening ratio  $K_{op}/K_{\text{max}}$  steadily decreased in both the thin and in the thick specimens as the cracks increased in size, decreasing the (predominantly elastic) residual ligament that tends to close them, it can be concluded that  $\Delta K_{\text{eff}} = K_{\text{max}} - K_{op}$  was *not* the FCG controlling driving force in this case. Indeed, since the loading conditions  $\{\Delta K, K_{\text{max}}\}$  were maintained constant during those tests, there is no doubt that  $\Delta K_{\text{eff}}$  steadily *increased* as the crack grew, because the decrease in  $K_{op}/K_{\text{max}}$  ratio is well beyond the (small) uncertainty of the measured data. Moreover, despite their slight lower R-ratios, note that the opening loads were a little bit higher along



the crack path in the thicker than in the thinner test specimens, or under plane strain instead of plane stress conditions, contrary to what could be expected beforehand. Finally, it should be mentioned that other limitations of the hypothesis “ $\Delta K_{eff}$  is the actual FCG driving force” are explored in [1].

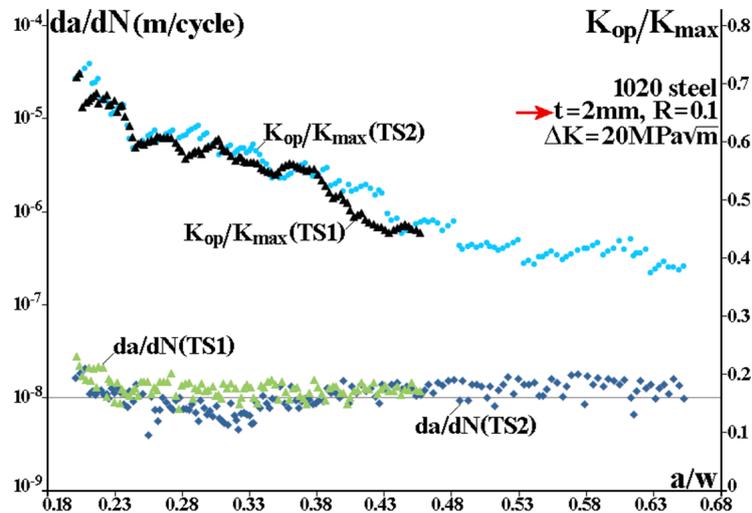


Figure 8: FCG rates  $da/dN$  and crack opening ratios  $K_{op}/K_{max}$  measured along the crack path in the thin DC(I) specimens ( $t = 2mm$ ), supposedly under  $pI-\sigma$  conditions.

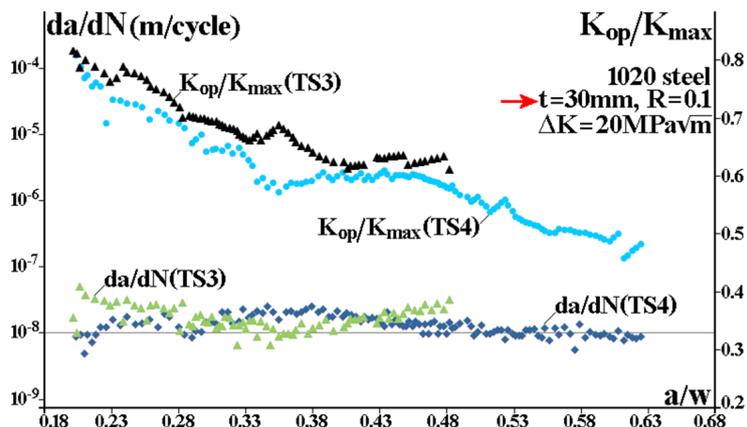


Figure 9: FCG rates  $da/dN$  and crack opening ratios  $K_{op}/K_{max}$  measured along the crack path in the thick DC(I) specimens ( $t = 30mm$ ), supposedly under  $pI-\varepsilon$  conditions.

## CONCLUSIONS

A long standing discussion about the actual fatigue crack growth driving force, which for many is the  $\{\Delta K, K_{max}\}$  pair whereas for many others is  $\Delta K_{eff}$ , remains more relevant than ever because it is a necessary condition to decide which model is the most appropriate to make FCG predictions under variable amplitude loading in practical applications. This study certainly does not settle this point, but it does present some easily reproducible results that clearly cannot be explained by the  $\Delta K_{eff}$  hypothesis.

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