Time and frequency domain models for multiaxial fatigue life estimation under random loading

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ABSTRACT. Engineering structures and components are often subjected to random fatigue loading produced, for example, by wind turbulences, marine waves and vibrations. The methods available in the literature for fatigue assessment under random loading are formulated in time domain or, alternatively, in frequency domain. The former methods require the knowledge of the loading time history, and a large number of experimental tests/numerical simulations is needed to obtain statistically reliable results. The latter methods are generally more advantageous with respect to the time domain ones, allowing a rapid fatigue damage evaluation. In the present paper, a multiaxial criterion formulated in the frequency-domain is presented to estimate the fatigue lives of smooth metallic structures subjected to combined bending and torsion random loading. A comparison in terms of fatigue life prediction by employing a time domain methods, previously proposed by the authors, is also performed.

KEYWORDS. Multiaxial fatigue. Critical plane approach. Random loading. Frequency-domain. Winde-band spectrum.

INTRODUCTION

The design of engineering structures subjected to cyclic loading characterized by randomly varying amplitudes is a complex and critical issue, which becomes even more complex in the case of multiaxial loading. In such a case, the procedures usually employed in the fatigue assessment of structural components are formulated in time domain or, alternatively, in frequency domain. The former procedures usually represent a generalisation of their counterparts for constant amplitude loading, by introducing a cycle counting method (e.g. rainflow method) and a damage model (e.g. the Miner rule) [1-5]. Some criteria present specific cycle counting methods to resolve multiaxial loading histories into individual cycles (e.g. see Refs [2, 6]). The knowledge of the time histories of the local stress or strain tensor components, experimentally measured on the structural component, is required, and many records are needed in order to obtain reliable statistical parameters of the loading process. Frequency-based procedures, instead, require the knowledge of cycle distribution of random loading from a statistical point of view: as a matter of fact, starting from the (PSD) power spectral density function of local stress or strain tensor components, an estimation of damage is directly obtained [7-10].

In the present paper, a stress-based critical plane criterion formulated in the frequency-domain is proposed to evaluate the fatigue lives of smooth metallic structures subjected to multiaxial random loading. It consists of the following three steps: (i) definition of the critical plane; (ii) PSD evaluation of an equivalent normal stress; (iii) determination of fatigue life. More precisely: (i) the critical plane is proposed to be dependent on the PSD matrix of the stress tensor; (ii) on such a verification plane, the PSD of an equivalent stress is defined by a linear combination of the PSD functions of both the normal stress and shear stress, the latter projected along the direction that maximises the variance of such a stress; (iii) the PSD of the above equivalent stress is used to estimate damage, and hence to determine fatigue life of the structural component via the Tovo-Benasciutti method [11]. The frequency-based criterion being presented is applied to combined bending and torsion random fatigue test data [12], which are also compared with the results determined by employing the time-domain criterion proposed in Refs [4,5].

CRITICAL (VERIFICATION) PLANE ORIENTATION

The verification plane orientation, named critical plane, is here assumed to be dependent on the PSD matrix of the stress vector [13], as is explained in the following.

Let us assume that: (i) the random features of the stress tensor, $\mathbf{s}_{xyz}(t)=\{s_1, s_2, s_3, s_4, s_5, s_6\}^T$, defined with respect to the fixed frame PXYZ (being P a point of the structural component), can be described by a six-dimensional ergodic stationary Gaussian stochastic process with zero mean values; (ii) the corresponding PSD functions are known. The coefficients of the PSD matrix, $\mathbf{S}_{xyz}(\omega)$, are given by:

$$S_{i,j}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} R_{i,j}(\tau)e^{-i\omega\tau}d\tau \quad i,j=1,\ldots 6 \tag{1}$$

where $R_{i,j}(\tau)$ are the auto- and cross-correlation functions and ω is the angular frequency.

The PSD matrix with respect to a rotated coordinate system PX'Y'Z' can be defined by employing the $\mathbf{S}_{xyz}(\omega)$ matrix and a rotation matrix $\mathbf{C}$ depending on the three rotation Euler angles φ,θ,ψ. In such a coordinate system, the corresponding stress tensor components and PSD coefficients are indicated as $s_{i'}$ and $S_{i',j'}(\omega)$, with $i',j'=1,\ldots 6$.

The above axes are made to vary as follows: the direction Z' (defined by the angles φ,θ) is such that $s_{3'}$ experiences the maximum in a statistical sense, according to Davenport [14]:

$$E\left[\max_{0\le t\le T} s_{3'}(t)\right]\cong\sqrt{\lambda_0}\,\sqrt{2\ln(\nu_0^+ T)+\frac{0.5772}{\sqrt{2\ln(\nu_0^+ T)}}} \tag{2}$$

being λ_0 the spectral moment of order 0 of the PSD function $S_{3',3'}$, ν_0^+ the expected rate of mean zero-upcrossings of $s_{3'}$ and T the observation time interval.

The Y' axis (defined by the angle ψ) is made to vary in order to maximize the variance of $s_{6'}$:

$$\max_{0\le\psi\le 2\pi}\left[\sigma^2_{6',6'}\right]=\left[\max_{0\le\psi\le 2\pi}\int_{-\infty}^{+\infty} S_{6',6'}(\omega,\psi)\,d\omega\right] \tag{3}$$

The directions Z' and Y' are regarded as the averaged principal directions $\hat{1}$ and $\hat{3}$, respectively.

Then, the normal to the critical plane, w , is defined in the $\hat{1}\,\hat{3}$ plane by the off-angle δ (clockwise rotation), function of the ratio between fully reversed shear fatigue limit and normal stress fatigue limit:

$$\delta=\frac{3\pi}{8}\left[1-\left(\frac{\tau_{af,-1}}{\sigma_{af,-1}}\right)^2\right] \tag{4}$$

PSD EVALUATION OF THE EQUIVALENT NORMAL STRESS

Let us define the PSD matrix with respect to a coordinate system Puvw, where u and v belong to the critical plane and w is the normal to the critical plane. In such a coordinate system, the corresponding stress tensor components and PSD coefficients are indicated as $s_{i''}$ and $S_{i'',j''}(\omega)$, with $i'',j''=1,\ldots 6$. The axes u and v are made to vary in the critical plane in order to maximize the variance of $s_{6''}$:

$$\max_{0\leq\gamma\leq 2\pi}\left[\sigma^2_{6'',6''}\right]=\left[\max_{0\leq\gamma\leq 2\pi}\int_{-\infty}^{+\infty}S_{6'',6''}(\omega,\gamma)\,d\omega\right] \tag{5}$$

being γ a counterclockwise rotation about the w -axis.
In order to reduce the multiaxial stress state to an equivalent unixial stress state, we propose to determine an equivalent PSD function through the following linear combination:

$$S_{eq}=S_{3'',3''}+\left(\frac{\sigma_{af,-1}}{\tau_{af,-1}}\right)S_{6'',6''} \tag{6}$$

FATIGUE LIFE EVALUATION FOR RANDOM LOADING

Let us consider the equivalent PSD function related to an equivalent unixial stress state, that is, to a one-dimensional stochastic process. In such a case, the expected fatigue damage per unit time, $E[D]$, may be evaluated by employing the following linear cumulative damage rule:

$$E[D]=\nu_a C^{-1}\int_0^{+\infty}s^k p_a(s)ds \tag{7}$$

being ν_a and $p_a(s)$ the expected rate of occurrence and the marginal amplitude distribution of the counted equivalent stress cycles, respectively, whereas k and C are the parameters of the normal stress S-N curve.
Note that damage estimation depends on the algorithm used to count loading cycles, that is, it is related to the method adopted to estimate the marginal density $p_a(s)$. Let us consider the Rain-Flow Counting (RFC) procedure [15]. For RFC methods, an analytical solution for $p_a(s)$ is not available in the literature and, therefore, Tovo and Benasciutti [11] addressed the problem of the RFC damage estimation as the search for the proper intermediate value between the lower and the upper bounds of $E[D_{RFC}]$.
By taking as counting variable an equivalent uniaxial stress having the PSD function S_{eq} proposed in Eq.(6), the expected fatigue damage $E[D_{RFC}]$ and the fatigue life T_{cal} are hereafter evaluated. In particular, by considering a critical damage equal to the unity, the calculated fatigue life, T_{cal}, is:

$$T_{cal}=\frac{1}{E[D_{RFC}]} \tag{8}$$

EXPERIMENTAL APPLICATIONS

The criterion proposed is hereafter applied to some results of fatigue tests on round specimens made of 10HNAP steel, subjected to a combination of random proportional bending and torsion [12]. Such a steel presents a fine-grained ferritic-pearlitic structure, and its mechanical properties are as follows: tensile strength R_m = 566 MPa, yield stress R_e = 418 MPa, Young modulus E = 215 GPa, Poisson ratio ν = 0.29. The adopted fatigue properties are: normal stress fatigue limit (under fully reversed bending) σ_{af} = 358.0 MPa at $N_0=1.282\times10^6$ cycles [12], shear stress fatigue limit (under fully reversed torsion) τ_{af} = 182.0 MPa at $N_0=1.282\times10^6$ cycles [12], inverse slope of normal stress S-N curve (under push-pull) k = 9.82 [4] (for σ_{af} = 358.0 MPa at $N_0=1.282\times10^6$ cycles C = 1.54(10)^{31} MPa$^{9.82}$).
Stationary and ergodic random loading applied to the above specimens presents zero expected value, normal probability distribution and wide-band frequency spectrum (0-60 Hz). The high-cycle fatigue tests here examined are related to two combinations of proportional torsional, $M_T(t)$, and bending, $M_B(t)$, moments, namely: 21 specimens for $\alpha = \pi/8$, and 14 specimens for $\alpha = \pi/4$ (Fig. 1). For each specimen, the fatigue life T_{exp} is experimentally determined.

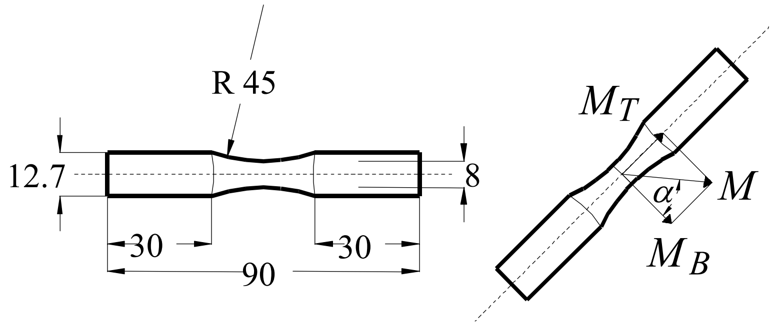

Figure 1: Fatigue tests on round specimens (10HNAP steel): torsion and bending.

The 35 specimens tested under multiaxial loading are analysed in the following by employing the actual experimental loading histories, which are characterised by a sampling frequency of 266.67 Hz and a total duration of 184.32 s [12]. Then, the theoretical procedure presented in the previous Sections is applied to such experimental data, and the fatigue life T_{cal} is computed. The comparisons between experimental data and theoretical evaluations are illustrated in Figs 2 and 3, where the solid line indicates $T_{cal} = T_{exp}$, the dashed lines correspond to T_{cal} / T_{exp} equal to 0.5 and 2 , and the dash-dot lines correspond to T_{cal} / T_{exp} equal to 0.3 and 3. A good agreement for the fatigue loading examined can be observed.

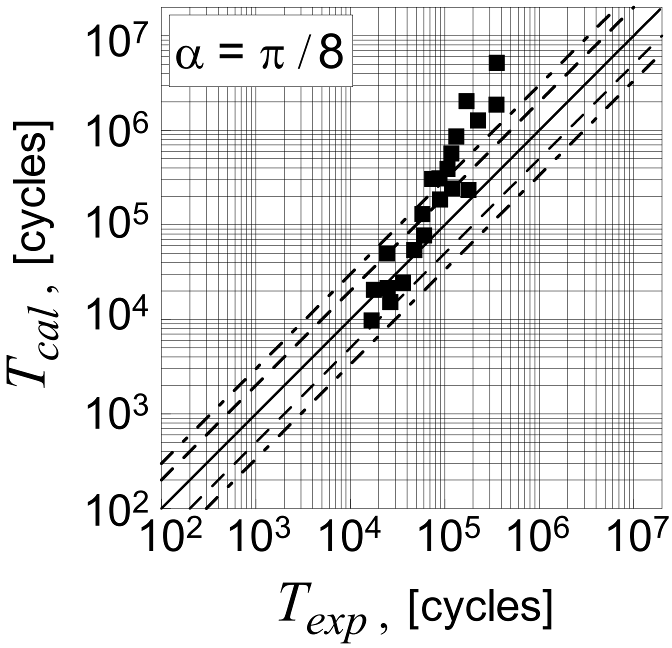

Figure 2: Comparison between experimental and theoretical fatigue lives ($\alpha = \pi / 8$).

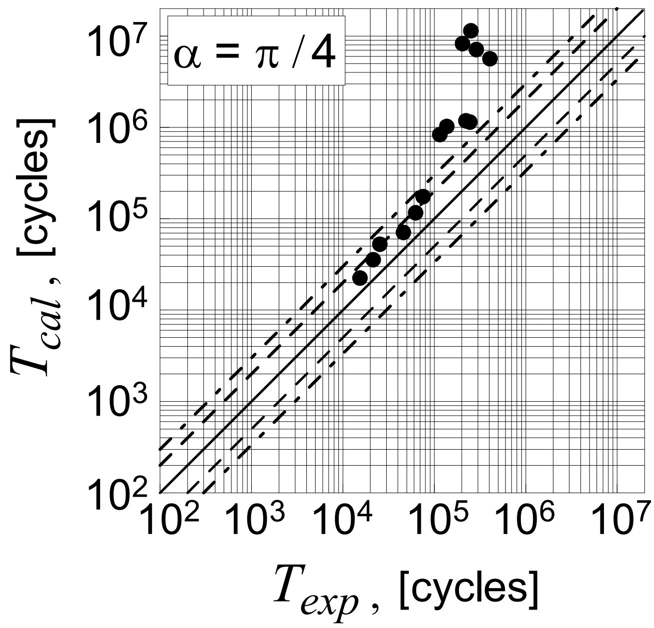

Figure 3: Comparison between experimental and theoretical fatigue lives ($\alpha = \pi / 4$).

The comparisons presented in Figs 2 and 3 seem to be quite satisfactory, with 63% of the fatigue life calculation results included into the scatter band with coefficient 2, whereas 66% of such results are included into the scatter band with coefficient 3.

The theoretical results based on the frequency domain approach are also compared with those determined by the time-domain criterion proposed in Ref. [4] (Figs 4-5). Note that the results here presented based on the time-domain criterion (based on a non-linear damage accumulation rule) are slightly different from those shown in Ref. [4], due to the different value adopted for the normal stress fatigue limit (in Ref. [4], σ_{af} = 225.0 MPa related to push-pull loading was adopted).

It can be observed that better results are derived by applying the new formulation, which has also the capability to be more computationally-efficient than that based on the time domain.

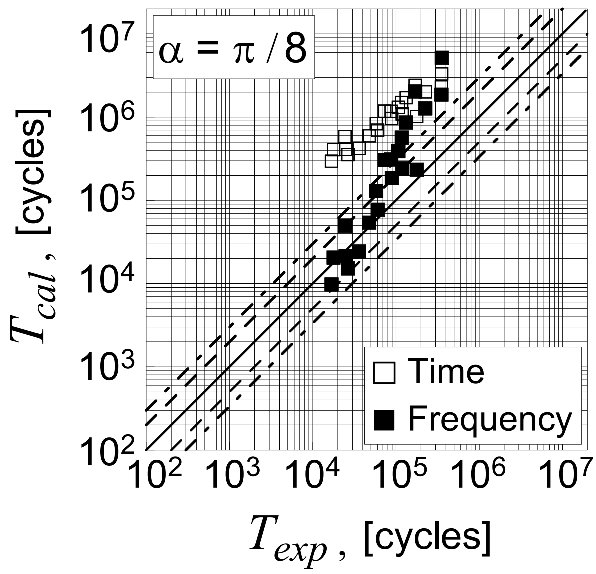

Figure 4: Comparison between experimental and theoretical fatigue lives ($\alpha = \pi / 8$), the latter determined by employing: the frequency domain criterion here proposed (see solid symbols) and the time domain criterion reported in Ref. [4] (see hollow symbols).

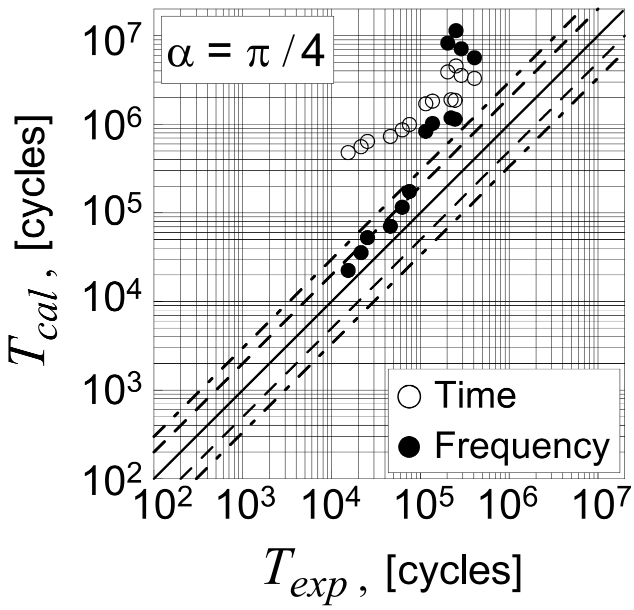

Figure 5: Comparison between experimental and theoretical fatigue lives ($\alpha = \pi / 4$), the latter determined by employing: the frequency domain criterion here proposed (see solid symbols) and the time domain criterion reported in Ref. [4] (see hollow symbols).

CONCLUSIONS

A frequency domain criterion is presented to evaluate the fatigue life of a smooth metallic structure subjected to multiaxial random loading. The critical plane orientation is determined through the PSD matrix of the stress tensor. Then, an equivalent PSD function is defined and processed through a damage model in order to determine the fatigue life of the structural component being examined. The comparison between theoretical and experimental results appears to be quite satisfactory for the cases analysed.

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