Focussed on: Fracture and Structural Integrity related Issues

Numerical modelling of reinforced concrete beams with fracture-plastic material

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ABSTRACT. This paper describes the use of models of fracture-plastic materials for reinforced concrete in numerical modelling of beams made from reinforced concrete. The purpose of the paper is to use of a model of concrete for modelling of a behaviour of reinforced concrete beams which have been tested at the University of Toronto within re-examination of classic concrete beam tests. The original tests were performed by Bresler-Scordelis. A stochastic modelling based on LHS (Latin Hypercube Sampling) has been performed for the reinforced concrete beam. An objective of the modelling is to evaluate the total bearing capacity of the reinforced concrete beams depending on distribution of input data. The beams from the studied set have longitudinal reinforcement only. The beams do not have any shear reinforcement. The software used for the fracture-plastic model of the reinforced concrete is the ATENA.

KEYWORDS. Concrete; Beam; Bearing Capacity; Reinforced; Non-linear Analysis; Fracture-plastic.

INTRODUCTION

Concrete ranks among modern building materials. It is used, in particular, in bearing structures in structural and civil engineering. These are, for instance, beams, walls, slabs, shell or volume enclosing sections. As the concrete is very specific, extensive research has been carried out. For instance, the concrete behaves differently if subject to tension or if subject to compression. For this reason, a non-linear analysis should be used in calculations.

More intensive use of the concrete and the need to carry out a detailed analysis have resulted in availability of many source documents and computational models for the concrete. Creation of such source documents have been among topics addressed by the International Federation for Structural Concrete (CEB-FIB) which issues recommendations and publications that are regarded as the starting point for creation of standards. In European standards, the recommendations of Model Code 1990 [3] are taken as a basis for designing of concrete and reinforced concrete structures. The latest recommendation is Model Code 2010 [4] which mentions several approaches applicable to the non-linear analysis.

This paper aims at validating the use of one of such approaches to the non-linear analysis. Because the design standards require that uncertainty should be considered in input data, the non-linear analysis is supported by the stochastic modelling which makes it possible to describe better the real behaviour and bearing capacity of the structure.

MATERIAL MODELS OF THE CONCRETE

In order to choose the model of the concrete correctly, it is necessary to consider the purpose and area where the concrete should be used because the concrete models require different data and accuracies of the models are different. This is, for instance, the case of details in the computational model. In most of advanced models the following is true: the more precise is the model, the higher are the demands in terms of quantity and quality of the complex data, this making, in turn, the computational process more demanding. A non-linear analysis of the concrete and reinforced concrete structures is often combined with other types of the task. Such tasks could, for instance, simulate dynamic loads or earthquakes [14]. Constitutive models of the concrete are implemented in several software applications [10], [18] and [21]. [1] and [21] deal also with the modelling and analysing of the concrete structures - they describe possible approaches to the behaviour of concrete elements and concrete models. [5] and [22] provide a good summary of the topic under study. One of advanced models - Fracture Plastic Material [9] in ATENA [6] - has been chosen for numerical modelling of the behaviour of the reinforced concrete beam. A predecessor of this concrete model is SBETA which is among widely-used models. The numerical analysis of stress and deformation in concrete structures uses the Finite Element Method [20].

STOCHASTIC MODELLING

In numerical analyses it is recommended to consider, in some cases, the random character of input data. The most common random inputs are material properties of the concrete. This is, for instance, the compression strength, tensile strength or modulus of elasticity of the concrete. In those cases, it is possible to use the stochastic modelling. Responses of the structure are calculated for individual sets of stochastic input parameters which describe uncertainty of input data [7] and [8]. The input values should be properly described, e.g. with a mean value, standard deviation, or type of distribution.

The analysis output are processed typically in histograms. Stochastic modelling is performed in FReET [17] which is a LHS software application. LHS (Latin Hypercube Sampling) and possible use are described in an theoretical manual [7] and user manual [8]. FReET is compatible with ATENA. The both software applications are supplied together as SARA. When selecting the random variables, it is recommended to follow JCSS [12] and ISO [11] and use a correlation matrix. The number of simulations depends on complexity of the task. Typically, it takes tens of hours to calculate the task.

EXPERIMENTS

More than sixty years ago one of the first important scientific work focused on the concrete appeared. The authors was Bresler-Scordelis [2] and his works has been supported by several experiments which have been properly documented. These were three-point bending tests of the reinforced concrete beams. The beam span and reinforcement were different. The work dealt with shear failures and total bearing capacity of the beams. The tests were used later for validation of several papers and recommendations. Twelve beams in four series were tested. Each series had a specific reinforcement and span. The shape was rectangular. The basic length of the beam was 3.66, 4.57 or 6.4 mm. Each beam was ca. 552 mm long. Fig. 1 shows principles of the experiment. More details about the test and numerical calculations are provided in [2].

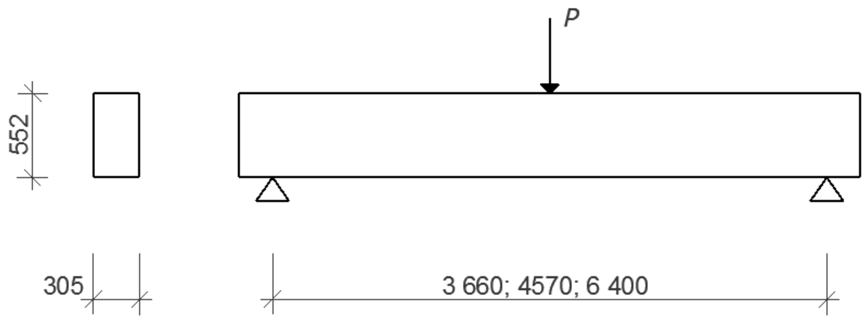

Figure 1: Scheme of the experiment.

The data obtained in the experiments were used for the testing software at the University of Toronto. The goal was to repeat the Bresler-Scordelis's tests of the reinforced concrete beams. The number, size and reinforcement of the beams were as close as possible and in line with current capabilities of the equipment. Differences in production dimensions were very little.

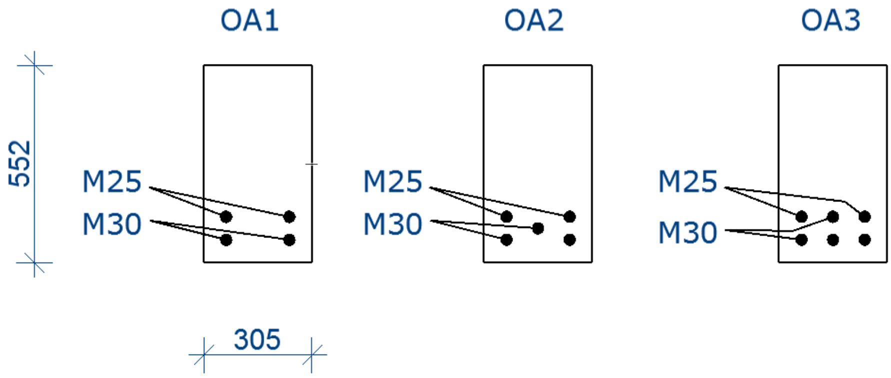

Figure 2: Cross-section of the beams.

Again, the purpose was to describe the behaviour of the reinforced concrete beams during the loading process and to determine collapse of the structure. The experiments were also used for the FEM numerical modelling [20]. Results of the experiments and numerical modelling are described in [19].

Bar size	Area [mm²]	f_y [MPa]	f_u [MPa]	E_s [MPa]
M25a	500	440	615	210000
M25b	500	445	680	220000
M30	700	436	700	200000

Table 1: Material properties of steels.

Beam number	L [mm]	*Span* [mm]	Bottom steel
OA1	4100	3660	2 M30, 2 M25
OA2	5010	4570	3 M30, 2 M25
OA3	6840	6400	4 M30, 2M25

Table 2: Detail of beams.

Beam number	f_c [MPa]	E_s [MPa]	f_{sp} [MPa]
OA1	22.6	36500	2.37
OA2	25.9	32900	3.37
OA3	43.5	34300	3.13

Table 3: Material properties of concrete.

The beams identified as OA were chosen for the numerical modelling in this paper. They are shown in Fig. 2. The beam is reinforced with the reinforcement identified as M25 and M30. The material properties of steel are described in Tab. 1. An elastic-plastic model of steel with reinforcement was chosen for the numerical modelling. M25b was used as a reinforcement for the beams 1 and 3, while the beam 2 was reinforced with steel identified as M25a. More details about dimensions and reinforcement of the beam are in Tab. 2. Original properties of concrete are described in Tab. 3. Specific features were calculated in ATENA [10] using the recommended values.

It follows from the test that the collapse in the beams without shear reinforcement occurs as a diagonal-tension failure. This is similar as in tests performed by Bresler-Scordelis. Examples of the failures are shown in [19]. This also shows the process of testing and the collapse.

NUMERICAL MODELLING

The numerical modelling was performed in ATENA [10] which uses FEM as a basis. That software includes a number of constitutive models of the concrete. It was decided to use a fracture-plastic material model for concrete and 2D computational models for the numerical analyses. Considering the numerical calculations performed in [19], very similar calculation models of the beams were created. The calculation model is a regular mesh of four-node finite elements. The finite elements in the concrete form a grid: 16x46, 16x56 and 15x66. Because the non-linear analysis depends much on the modelled boundary conditions, supports and loading plate from a linear elastic materials were also modelled.

The calculation models were formed by a symmetric half of the real beams. In order to solve a system of non-linear equations, the Newton-Raphon method and deformation loads were chosen in ATENA [10]. Fig. 3 shows the final calculation models. The reinforcement was included into the calculation model as a smeared reinforcement.

Beam OA1

Beam OA2

Beam OA3

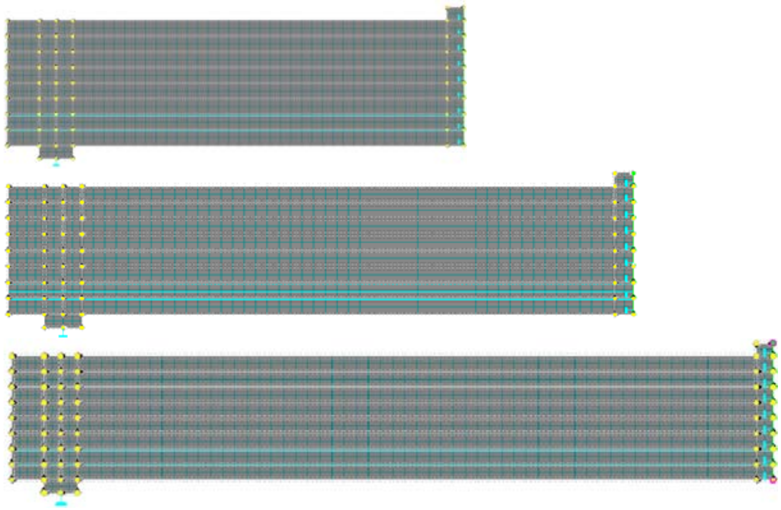

Figure 3: Computer models of beam (OA1, OA2, OA3).

The analysis focused on development of cracks during the loading, maximum bearing capacity P_u and deformation w_u. The calculations have been performed in two alternatives in Tab. 4 a 5. The first alternative includes all properties of the concrete which are mentioned in Tab. 3, while the second alternative takes the compressive strength of the concrete as a basis. The basis is the recommended values for standard concrete. The remaining values are calculated in ATENA [10]. The software calculates also the tensile strength and modulus of elasticity of concrete. Those data are not so frequently available and their values are rather distributed in practical engineering.

Beam number		Ultimate load			Midspan deflection	
	$P_{u,\,Test}$ [kN]	$P_{u,\,Calc}$ [kN]	$P_{u,\,Test}/P_{u,\,Calc}$	$w_{u,\,Test}$ [mm]	$w_{u,\,Calc}$ [mm]	$w_{u,\,Test}/w_{u,\,Calc}$
OA1	331	332	1.00	9.1	6.8	1.35
OA2	320	358	0.89	13.2	13.2	1.00
OA3	385	374	1.03	32.4	30.6	1.06
		Mean	0.97		Mean	1.14

Table 4: Comparison of the numerical calculations and experiments – alternative 1.

Beam number	Ultimate load			Midspan deflection		
	$P_{u,\,Test}$ [kN]	$P_{u,\,Calc}$ [kN]	$P_{u,\,Test} / P_{u,\,Calc}$	$w_{u,\,Test}$ [mm]	$w_{u,\,Calc}$ [mm]	$w_{u,\,Test} / w_{u,\,Calc}$
OA1	331	315	1.05	9.1	6.7	1.37
OA2	320	308	1.04	13.2	11.2	1.18
OA3	385	334	1.15	32.4	24.5	1.32
		Mean	1.08		Mean	1.29

Table 5: Comparison of the numerical calculations and experiments – alternative 2.

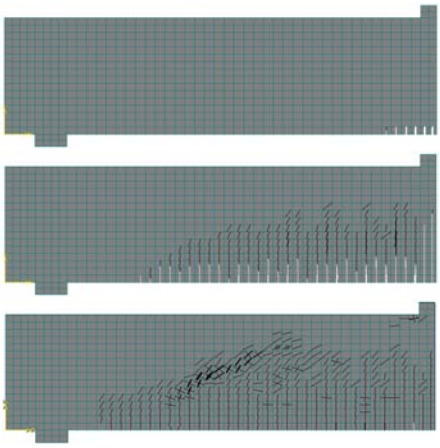

Figure 4: Failure in a beam, OA1.

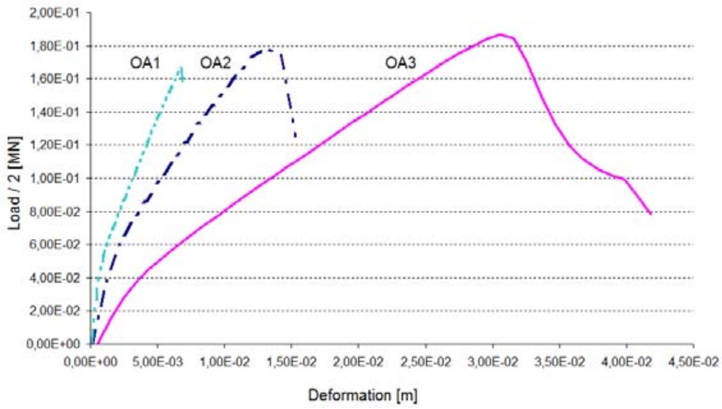

Figure 5: Load – displacement diagram for beams OA

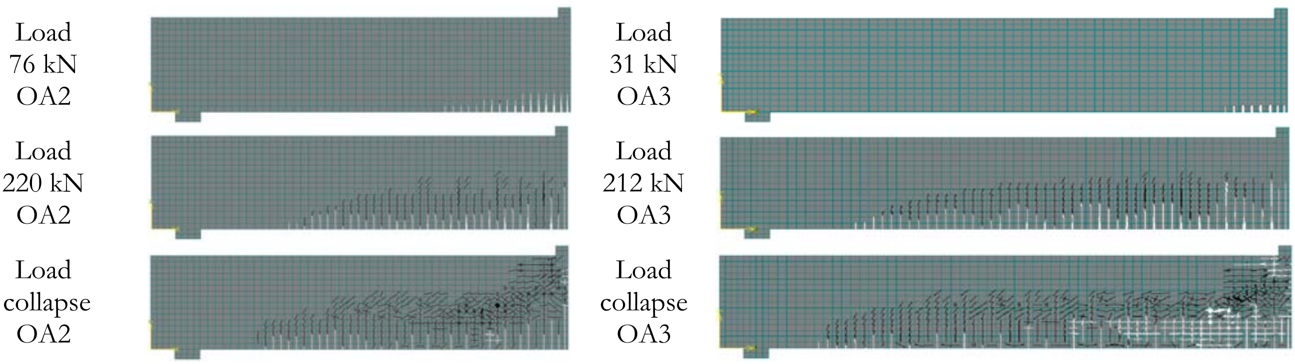

Figure 6: Failure in a beam. OA2 (left) and OA3 (right).

Fig. 5 shows the final comparison of work diagrams for the beams obtained in the numerical calculation. Fig. 4 and 6 show three typical loading conditions where cracks develop in each beam. The first condition is development of cracks next to the lower edge of the beam. The second condition is development of tensile cracks along the lower edge immediately before creation of a shear crack. The third condition is a collapsing beam.

STOCHASTIC MODELLING

Behaviour of beams under load was analysed in detail in a stochastic modelling. The objective was to find out impacts of some input data which enter the calculation as a histogram onto the total bearing capacity. The stochastic modelling was carried out using LHS as a method and FReET [7] as a software application. Statistic parameters were described using the recommendations specified in JCSS [12] and ISO [11]. Tab. 6 and 7 list the chosen histogram parameters and COV for the input variables of concrete and steel. The initial value was the compressive strength of the concrete. Tab. 8 shows the correlation matrix used for the concrete in the stochastic modelling.

Input	E_c	f_c	f_t	G_f
Distribution	Lognormal	Lognormal	Weibull	Weibull
COV	0.15	0.10	0.18	0.2

Table 6: Material properties in the stochastic modelling - concrete.

Input	Fy	Fu
Histogram	Lognormal	Lognormal
COV	0.05	0.05

Table 7: Material properties in the stochastic modelling - steel.

Input	E_c	f_c	f_t	G_f
E_c	1	0.7	0.9	0.5
f_c	0.7032	1	0.8	0.9
f_t	0.8972	0.7987	1	0.6
G_f	0.5021	0.8991	0.6014	1

Table 8: Correlation matrix of concrete.

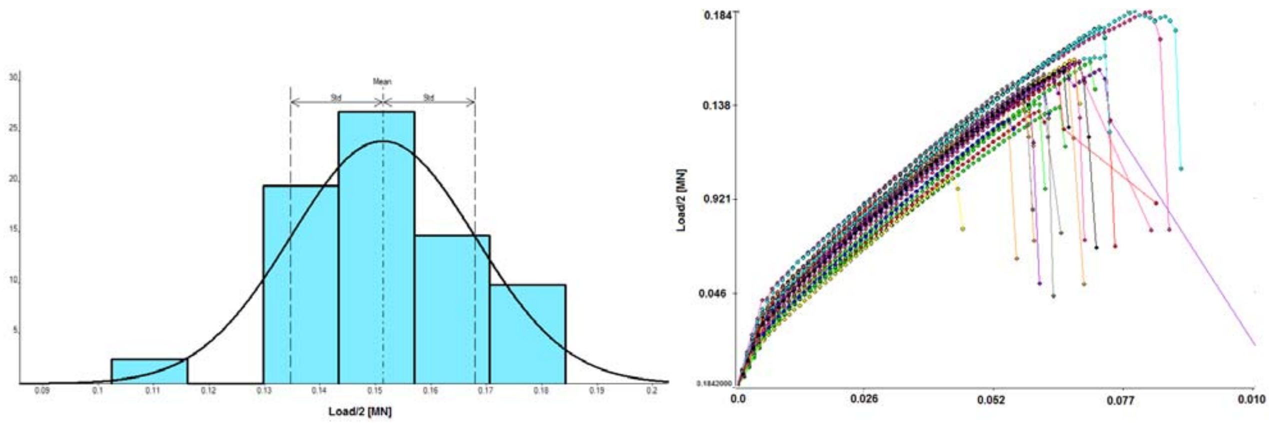

Figure 7: Final histogram, estimate - ultimate load.

DISCUSSION

The University of Toronto tested the beams without shear reinforcement. Three basic phases of load can be identified in working diagrams and photos published in [19]. It follows from the comparison of the working diagrams, photos and numerical calculations that these are the three basic phases of the loading process.

At the beginning of the loading process, cracks appear at the lower edge in the middle of the span. Then, shear cracks appear. They become dominant, until the beam collapses. The collapse occurs fast. In case of beams with a big span (OA2 and OA3), the crack is located towards the loading point and is almost horizontal. The crack propagates in places where the reinforcement is located.

In each numerical calculation the final way of collapse in a beam was same as in the experiment. These were diagonal-tension failures. The comparison of the total bearing capacity obtained in calculations and that obtained in experiments shows very good correlation for both the first and second alternatives. The medium values of $P_{u,\,Test} / P_{u,\,Calc}$ are 0.97 and

1.08. In case of the maximum deformation the correlation was not so good. The mean value of $w_{u,\,\mathrm{Test}}\,/w_{u,\,\mathrm{Calc}}$ for the first and second alternatives is 1.14 and 1.29, respectively. The conclusion, however, is that the results are satisfactory with respect to the input data. It follows from the results that the worst correlation was obtained for the OA1 beam for which the stochastic analysis was performed.

Fig. 7 shows the final estimate of the histogram for the total bearing capacity. It follows from the evaluation of the histogram data and comparison with results of the numerical analysis that the bearing capacity ranges in a rather interval from 204.8 kN until 368.4 kN, provided that normal distribution is assumed. The resulting histogram is assumed normal distribution, the mean value is 302.5 kN.

It should be also pointed out that the results could be influenced by the modelling of supports and loads, by the size of the loading step or by the size of the finite element.

CONCLUSION

The numerical analysis indicate that the calculations performed using the fracture-plastic material [9] for concrete describe the loading process of reinforced concrete beams without shear reinforcement very well, the final bearing capacity correlating well with the experiments. The maximum deformation of the beam also proved good correlation between the calculation and experiment. The calculation was, however, too sensitive to input properties of the concrete. This was the evaluation of results of the stochastic modelling where the final deformation and bearing capacity lied in a rather big interval. Calculations which used the compressive strength of concrete only described well the real behaviour of a beam, with respect to the quantity of estimated input data. The results of the numeric calculation and stochastic modelling can be used in calculation in line with the proposed standards.

With this procedure, it is, however, necessary to use a suitable the global safety limits for the design values. The authors will focus now on the stochastic modelling and probabilistic methods which include, for instance, [13], [15] and [16]. The reason is that the stochastic calculations take much time and this makes them inconvenient for wide use and for drawing conclusions.

In general, the model of concrete describes well the total bearing capacity of a concrete beam and development of failure during loading. The reason for difference between the experiments and numerical analyses is probably approximation of specific parameters and uncertainty in input data.

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