A K_{eff} Function for Complex Mechanical Elements

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Abstract

The aim of this paper is to study fatigue crack propagation in a real mechanical component. Experimental fatigue tests have been carried out. Numerical finite element models have been developed to calculate the effective stress intensity factors, K_{eff}, *which consider all the propagation modes. In order to avoid making a large number of numerical models of a cracked element, a function of* K_{eff} *values versus the depth and the surface length was found. In this way it is possible to follow the evolution of the crack propagation. In order to have a reliable prediction of the growth of the crack it is necessary to consider all the fracture modes.*

Riassunto

Lo scopo del lavoro è studiare la propagazione di cricche di fatica in un elemento meccanico di geometria complessa. A tal fine sono state effettuate prove sperimentali. Inoltre lo sviluppo di differenti modelli ad elementi finiti ha permesso di calcolare un fattore di intensità degli sforzi effettivo K_{eff}, che considera l'influenza dei diversi modi di propagazione delle fratture. Per evitare di ripetere le analisi numeriche del componente per numerose lunghezze configurazioni della frattura, è stata definita una funzione analitica di K_{eff}, rispetto alla profondità della cricca ed una rispetto alla sua lunghezza superficiale. I risultati mostrano che per avere una previsione accurata della propagazione a fatica della cricca, è necessario considerare l'influenza di tutti i modi di apertura della frattura.

Introduction

A large number of in-service failure in structures and components in aerospace, civil and mechanical engineering can be attributed to the incipient growth of flaws subjected to cyclically varying stresses [1]. The total fatigue life, N_f, of a structure is the sum of the number, N_i, of cycles to nucleate a crack and the number, N_p, to propagate it to final failure:

$$N_f = N_i + N_p \tag{1}$$

The classical approach based on total fatigue life considers the crack initiation as a critical point, in fact most of high-cycle fatigue life is spent in the formation of an engineering-sized defect. This especially is true in smooth tests specimens, but if the mechanical component is complex it is even necessary to consider the mean stress effect, the effective stress concentration at notches, the variable amplitude loading, the multiaxial stresses and the environmental effects.

Due to the presence of notches the stresses are often so large that the material yields and the resulting fatigue bebaviour is characteristic of low-cycle fatigue. For this reason the number of propagation cycles becomes important with respect to the total fatigue life.

Moreover in safety critical structures it is assumed that defects exist in the material and these are taken into consideration during design. Fatigue life is evaluated in terms of the number of cycles required to propagate the largest undetected crack to failure; the knowledge of macroscopic crack propagation under realistic in service loading is essential to correctly design fail-safe structures.

The crack growth is well described by Paris equation [2], which correlates the crack growth rate to the stress intensity factor, K:

$$da/dN = C \cdot \Delta K^n \tag{2}$$

where C and n are experimentally determined characteristics of the material. In order to predict the crack growth it is necessary to know the value of K. In literature [3], [4], [5] it is possible to find the expression of K for a large variety of situations, most of which are, however, of the first mode opening type.

On the contrary in real structural components, due to the complex geometries and loading conditions, the fatigue cracks are often subjected to mixed-mode loading. Some authors [6], [7], [8] have proposed the following growth law:

$$da/dN = C \cdot (\Delta K_{eff})^n \tag{3}$$

where the effective stress intensity factor is defined for combined mode I and mode II loading. The authors also made a comparison with the experimental results and they obtained a satisfactory agreement. Tanaka [7] extended this propagation law to the case when all the loading modes are present, but he did not conduct experimental tests under these loading conditions.

In this paper, on the contrary, the authors show the experimental results obtained from fatigue tests carried out on a mechanical component, a crankshaft, with a complex geometry, where the cracks grow under three-dimensional stress state. In this situation in literature the expression of the stress intensity factor cannot be found and three-dimensional numerical models have also been constructed to calculate it.

In order to study the trend of the stress intensity factors the different models have different crack sizes. A large number of calculations have been carried out and the K_I, K_{II} and K_{III} values have been determined; a K_{eff} value is then found by using Tanaka's relation and by the numerical J-Integral values.

Experimental tests

The experimental tests were described in detail in previous works [9], [10].
Single cranks of marine Diesel engines were used for the fatigue tests. The crankpin diameter is equal to 130 mm. Crankshafts were constructed by using 35CrMo4 UNI 7874 steel.

The crankshafts were loaded with an eccentric alternate axial load P, which was applied by means of a mechanical device. The crankpin was subjected to a constant bending. The cracks originated in the most stressed zone near to the crank fillet. Crack propagation is measured by means of crack gages until complete rupture of the element.

In the crankshafts tested it was observed that growth of the fatigue crack begins in a plane normal to the direction of the maximum stress, inclined at 45° in relation to the load direction and propagates symmetrically with respect to the plane of symmetry.

Numerical analyses

The cracked crankshaft was finite element modelled for four different values of the crack length, b. The fracture shapes, shown in fig. 1, were, like the real ones, assumed to be elliptical and symmetrical with respect to the medium plane of the element. The crack dimensions schematized in the numerical models correspond to a number of cycles $N=385.000$, $N=465.000$, $N=540.000$ and $N=570.000$. The corresponding external crack lengths are shown in Fig. 1, $b=64$ mm, $b=90$ mm, $b=120$ mm and $b=135$ mm.

Particular care was taken to model the zone near to the crack; the "one quarter point" technique [11] was adopted to increase the precision of the calculation near the crack. A convergence study of the results was also carried out until the values of the stresses were found to be stable. The load case reproduces the experimental tests; further details of the finite element models are shown in [9], [10].

Two analytical functions of $K_{eff}(x)$ were defined by fitting the numerical results: the first considers the internal maximum depth a as x, the second considers the external crack length b as x. In this way it is possible to relate the values of the crack propagation rate dx/dN ($x=a$, b) to the values of the effective stress intensity factor K_{eff} and to predict the internal and external fatigue crack propagations along the crack front using a few numerical analyses.

Results

In [10] the trends of the numerically calculated normalised stress intensity factors k_i ($k_i = K_i \sigma_{nom} \sqrt{\pi a}$) are shown.

Besides the finite element analyses directly provide the J-integral values, obtained by using the virtual crack extension method. In fig. 2 the trend of J-integral along the crack fronts for all the cracks considered is shown. It is possible to observe that the J-values are substantially stable in the internal part of the crack front, while they increase near the free surface, where the effect of the curvature cannot be neglected.

By considering the following relation:

$$J = K^2_{effVC} \; / \; E' \tag{4}$$

(where E' is equal to E for plane stress condition and is equal to $E/(1-\nu^2)$ for plane strain condition) the corresponding effective stress intensity factors were calculated.

On the contrary the values of the stress intensity factors K_i were evaluated by extrapolating the nodal values obtained from the stresses to the crack tip.

The K_{effT} values are then determined by means of the relation [7]:

$$K_{effT} = [(K_I^4 + 8K_{II}^4)]^{1/4} \tag{5}$$

if the loading mode is a combination of model I and II, and:

$$K_{effT} = [(K_I^4 + 8K_{II}^4) + 8K_{III}^4/(1-\upsilon)]^{1/4} \tag{6}$$

if the loading mode is a combination of modes I, II and III.

In fig. 3 the trend of $K_{eff}(b)$ and $K_{eff}(a)$, calculated by fitting the data with a second order polynomial function (whose numerical expression is illustrated in Fig. 3) are shown; both K_{effT} and K_{effVC} are considered at the deepest point and at the free surface: it is possible to note that on the surface the two values are both similar to the values of K_I reported in [10], while inside the values are different.

In fig. 4b the trend of the external crack growth, calculated by integrating the eq.(3), by considering $C = 1.15 \cdot 10^{-12}$, $n = 3$ (dx/dN [m/cycle]; K_{eff} [MPa $\sqrt{\text{m}}$] and by using the two different definition of the effective stress intensity factor, is compared with the experimental external crack growth obtaining a good agreement in both cases. If we consider the internal propagation (see Fig. 4a), and we use the same values of C and n, the behaviour obtained by integrating the K_{effVC} crack propagation law is more realistic with respect to the experimental points found by marking the crack surface, while using K_{effT} lower crack growth rate are calculated.

Looking at the values of the previously defined effective stress intensity factor, it can therefore be noted that Tanaka's K_{eff} understimates the influence of the II mode, whose contribution becomes stronger in the internal crack propagation.

Conclusions

The growth of a crack in a mechanical component was experimentally and numerically studied. The stress state acting on the crack was multiaxial, therefore a K_{eff} value was defined in two different ways. The value of the stress intensity factors, K_i, was calculated by means of a finite element model. Several numerical models were developed with the aim of following the crack propagation and the K_{eff} values with respect to the maximum depth and the external crack dimension.

The definition of a K_{eff} function versus a and b enables not only fewer numerical models to be developed but also to follow the entire crack growth range.

The comparison with the experimental values shows that the external crack growth is mainly governed by the first stress intensity factor; on the contrary, the internal crack propagation is governed even by the other crack opening modes, that must be taken into account. The experimental evidence shows that by using a crack propagation law as a function of the K_{effVC} values accurate crack growth predictions can be obtained.

Symbols used

a	= crack depth
b	= crack length on the free surface
E	= Young's modulus of elasticity
$K_{I,II,III}$	= opening, shearing and tearing mode stress intensity factors
K_{effT}	= effective stress intensity factor according to the Tanaka's definition
K_{effVC}	= effective stress intensity factor calculated by the J-integral values
N	= number of cycles

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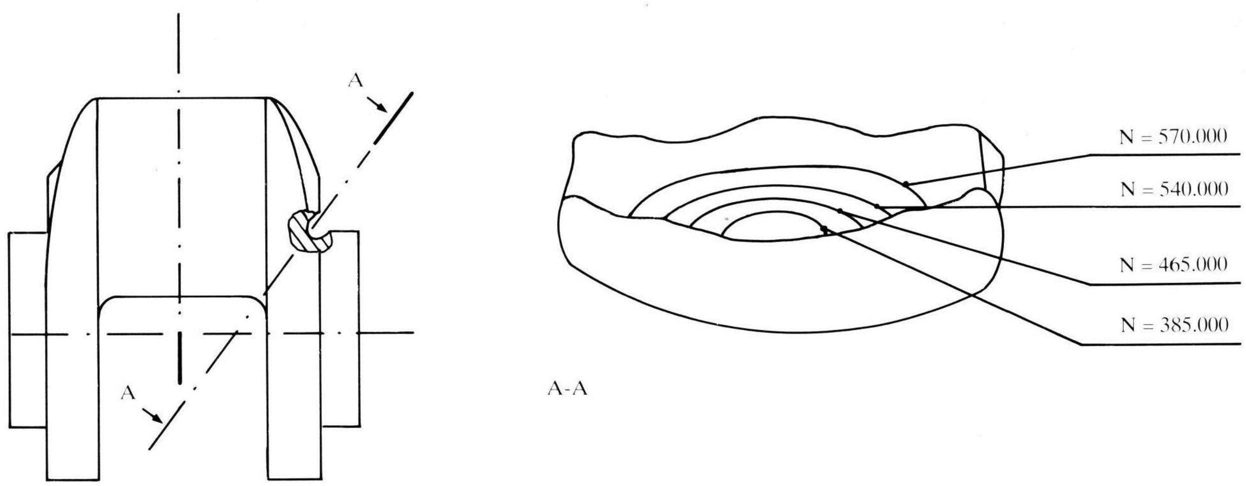

Fig. 1:
Scheme of the crankshaft and cracked section.
It is possible to not the elliptical crack fronts.

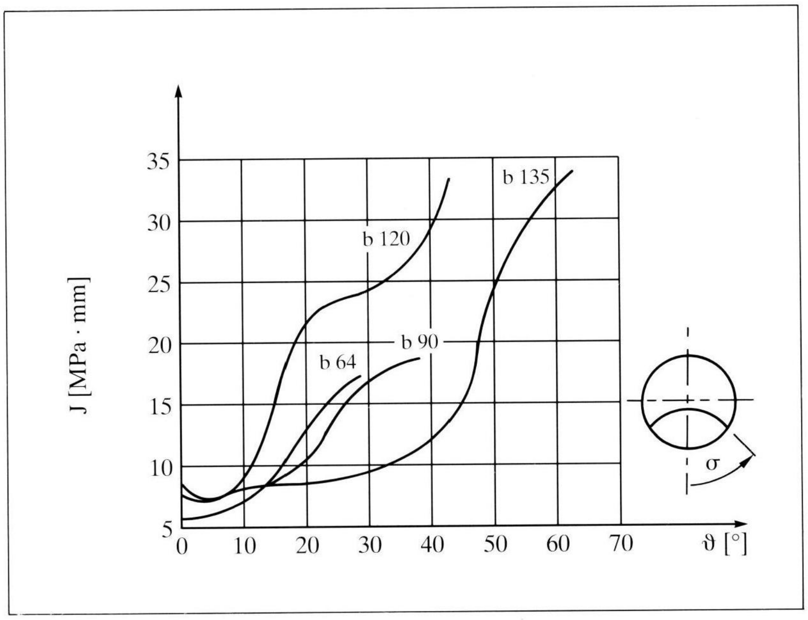

Fig. 2:
J-integral trend along the fruo crack fronts.

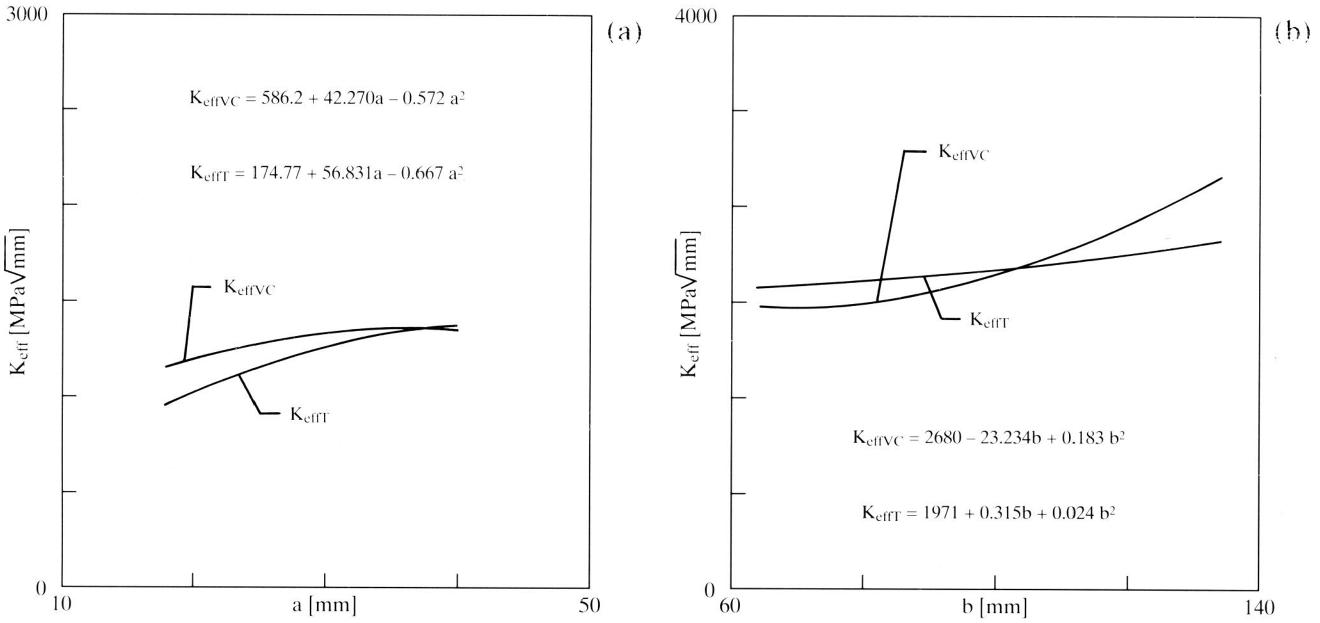

Fig. 3:
K_{effT} and K_{effVC} trend at the deepest point of the crack front (a) and on the free surface (b).

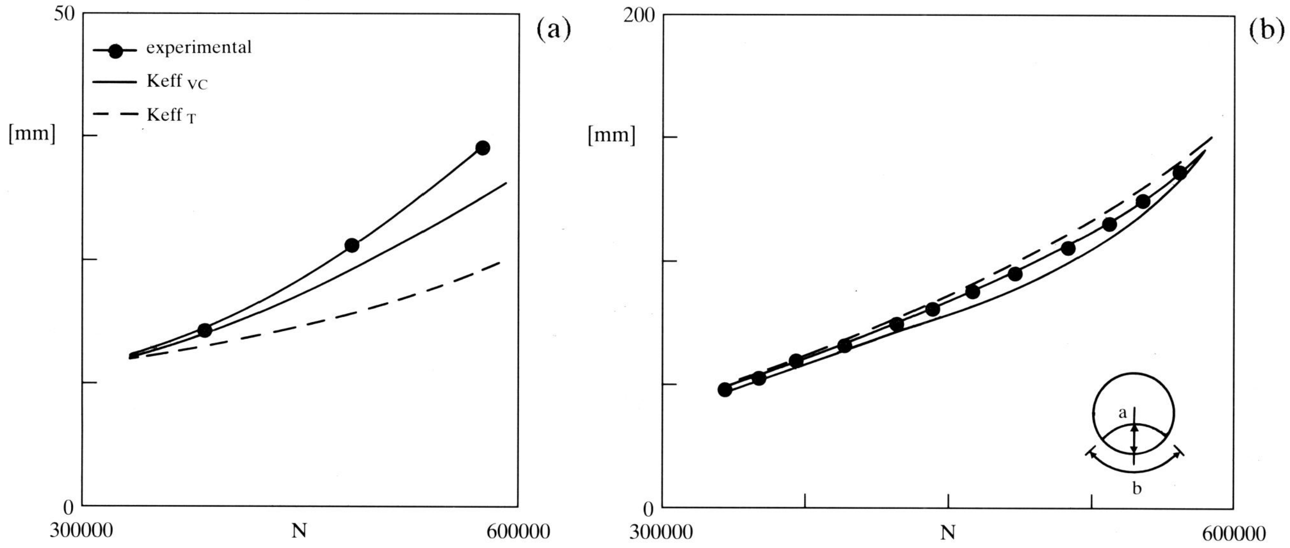

Fig. 4:
Experimental and predicted crack propagation at the deepest point of the crack front (a) and on the free surface (b).