Field Criteria for Failure Analysis: First Experimental Results on V-Notched Specimens *

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Abstract
Cast iron specimens with V-notches of different depths are axially loaded to verify if failure can be predicted by means of field criteria, like classic approaches of fracture mechanics. Test results are compared with the theoretical predictions achieved from the linear analysis carried out with a boundary element code. The results are then discussed considering the influence of the material non-linearities.

Riassunto
Si sottopongono a prove di trazione provette in ghisa grigia con intagli a V di diverse profondità, al fine di verificare se la rottura può essere prevista per mezzo di criteri di campo, seguendo un approccio analogo a quello classico della Meccanica della Frattura. I risultati delle prove sono confrontati con le previsioni teoriche ottenute dall'analisi lineare condotta con un codice di calcolo a Elementi di Contorno e sono poi discussi considerando l'influenza della non linearità del materiale.

Introduction

Failure analysis of notched machine elements is usually based on the stress concentration factor, which reduces the allowable strength of materials. This method cannot be reasonably used with sharp notches, because of the stress field singularity: in this case the theoretical stress concentration factor would rise to infinity while the material notch sensitivity would vanish. Besides mechanical elements often present sharp angle notches, especially $\pi/2$V notches, as spigots, keyways in shafts and hubs, gaskets seats of pressure vessels, etc.

Field criteria, already used in fracture mechanics, have proved to be suitable for failure analisys of cracked elements, where the stress singularity order is $0(1/\sqrt{r})$ [1]. The applicability of analogous criteria to singularities of different order, caused by re-entering corners, is less investigated.

The stress singularities in re-entering angles of plates have been studied analytically [2] and the stresses and displacements expressions are known in a polar reference with the origin in the tip. In these cases the stress intensity parameters for a finite region can be found numerically [3] and the first applications of field criteria to fatigue failure of V-notched elements are proposed in [4], [5] and [6].

By means of a boundary elements code [7], [8] and [9], in previous works the authors have brought a preliminary analysis of V-notched specimens.

Referring to the V notch of the specimens tested, the stress field is symmetrical with respect to the bisecting x axis (Fig. 1); in the neighborhood of the tip, the singularity of the elastic solution is dominant and in a polar r, θ reference can be generally represented by

$$\begin{cases} \sigma_x = \dfrac{K}{r^\alpha} \cdot \{(2+A)\cos(\alpha\theta) + \alpha\cos[(\alpha+2)\theta]\} \\ \sigma_y = \dfrac{K}{r^\alpha} \cdot \{(2-A)\cos(\alpha\theta) - \alpha\cos[(\alpha+2)\theta]\} \\ \tau_{xy} = \tau_{yx} = \dfrac{K}{r^\alpha} \cdot \{A\sin(\alpha\theta) + \alpha\sin[(\alpha+2)\theta]\}, \end{cases} \tag{1}$$

with $|\theta| \leq \dfrac{2\pi-\beta}{2}$, having placed

$$A = \cos[(2\pi-\beta)(1-\alpha)] + (1-\alpha)\cos\beta;$$

where α is the solution of the trascendental equation

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$$\frac{1}{1-\alpha} \sin\left[(2\pi-\beta)\ (1+\alpha)\right]-\sin\beta. \quad (2)$$

For $\beta = \pi/2$ the solution is $\alpha \sim .4555$.

The maxima of $\sigma_y(\theta)$ and $\sigma_x(\theta)$ are reached for $\theta = 0$ on x axis and are worth

$$\sigma_y(0) \simeq 2.383 \cdot \frac{K}{x^\alpha}$$
$$\sigma_x(0) \simeq 1.617 \cdot \frac{K}{x^\alpha} \quad (3)$$

The parameter K, defined above, is used to represent the stress intensity of the field. Such parameter is similar to the the stress intensity factor used in classic fracture mechanics but it is dimensionally different being related to the $0(r^{-\alpha})$ singularity due to a right angle re-entering corner. In [10] the stress intensity parameter K has been evaluated for specimens with notches of various depth. The significance of this parameter has been discussed relating to the specimen dimensions and the material behaviour: furthermore the disturbance due to the near boundary has been analysed and the notch depth corresponding to the maximum extension of the weakly disturbed zone has been determined. In [11] the perturbations of the singular stress fields due to geometric imperfections of the notch tip, microstructural non-homogeneities and material non-linearities, have been considered; besides a general applicability criterium of linear and non linear fracture mechanics has been established.

The present paper describes the experimental tests conducted on six sets of cast iron V-notched specimens.

Description of the tests

Six series of five specimens with two symmetrical V-notches with $\beta = \frac{\pi}{2}$ have been axially loaded; the specimens dimensions are shown in Fig. 1 and Table 1.

TABLE 1 - Specimen dimensions

Specimen	W [mm]	C [mm]	S [mm]	L [mm]
A	40	10	11	240
B	40	4	11	240
C	40	7	11	240
D	40	13	11	240
E	40	10	7	240
F	26	6.5	11	240

The material was a grey cast iron with tensile strength of 250MPa whose stress-strain curve is reported in Fig. 2. This curve was obtained by a tensile test on a specimen, without notches, equipped with strain gauges. The specimens were machined from $40 \times 250 \times 13$ bars obtained from the same cast to grant uniform chemical and microstructural characteristics. The chemical analysis results are shown in Table 2, while a 200x picture (Fig. 3) illustrates the metallurgical structure, obtained by etching with a 3% Nital solution. The metallographic structure has thin and sharp graphite lamellae uniformely distributed without a preferential orientation: being their dimensions between .12 and .25 mm the cast iron can be classified of the kind "grafite IA4" according to the 3775 UNI norm. A larger magnification observation shows a perlitic structure. The V-notch was milled by a sharp cutter. It showed a tip radius $r \simeq 0.1$ mm. Since the radius size is smaller than the material non-homogeneities, the theoretical stress concentration factor cannot have any effective significance. The test machine used is a Schenck Hydropuls 250 kN. The gripping was particulary careful to avoid bending effects due to misalignment between the upper and the lower grip; in particular, the first specimen tested has been equipped with strain gauges to improve the alignment procedure. The tests were stress controlled and the experimental data acquired by an analog-digital system.

TABLE 2 - Chemical composition of the cast iron used

C	Si	Mn	P	S	Ni	Cr	Cu	Mo	Al	Sn	Pb	As	Nb	Ti
3.42	2.05	0.60	0.025	0.120	0.050	0.130	0.170	0.015	0.003	0.016	tr.	0.004	0.004	0.009

Discussion of the results

All specimens failed in a brittle manner and fractures initiated at notch tips. The test results are shown in Table 3. Having referred the nominal stress $\bar{\sigma}$ to the specimen section of minimum area

$$\bar{\sigma} = \frac{F}{(W-2C) \cdot S}$$

the mean values of the nominal rupture stresses $\bar{\sigma}_R$, reported in Table 3 with their scattering, are evaluated as

$$\bar{\sigma}_R = \frac{F_R}{(W-2C) \cdot S}$$

being F_R the ultimate tensile load.

In the last column of the table are shown the predictions based on the field criterium in the linear elastic range (fracture occurs when the stress intensity parameter K reaches a critical value) being

$$\bar{\sigma}_{RKI} = \frac{F_{KI}}{(W-2C) \cdot S}$$

where F_{KI} is the axial load necessary to reach the value $K_I = 108.9 N/mm^{2-\alpha}$ (1), (3). The limit value of K_I is the failure value reached by the specimens A, assumed as reference because they have the maximum extension of the zone weakly disturbed by the boundary conditions [10]. The relation between the parameter K and the applied load F has been calculated by means of a widely tried boundary element code [8] considering a linear behaviour of the material.

TABLE 3 - Test results

Specimen	No. of repeated tests	$\bar{\sigma}_R$[MPa]	$\bar{\sigma}_{RKI}$[MPa]
A	5	189 ± 4.2	189
B	5	187 ± 3.5	189
C	5	183 ± 3.6	179
D	5	198 ± 4.8	216

(continue)

E	5	191 ± 3.0	189
F	4	202 ± 1.9	230

In Fig. 4 the experimental results $\bar{\sigma}_R$ are compared with the predictions $\bar{\sigma}_{RKl}$ and $\bar{\sigma}_{RKnl}$ (dashed curve) calculated taking approximately into account the non linearity of the material.

The approximate non-linear prediction is obtained by correcting the linear analysis results: in the surroundings of the fracture section the actual strains are supposed to be very closed to the elastic ones obtained with a Young modulus equal to the secant modulus of the experimental $\sigma(\varepsilon)$ curve at $\sigma = \bar{\sigma}$. In fact the linear analysis proves that the stress distribution is near the mean value $\bar{\sigma}$ in a great part of the section; just before the failure this will be even more true since the material behaves non-linearly and the $\sigma(\varepsilon)$ curve slopes down. The correction procedure is reported in Fig. 5. Besides supposing that very near the notch tip, in the process zone, just immediately before the failure, the strain field would be the same for all the specimens, it follows that the rate between the linear stress intensity parameter K and the secant elastic modulus would be costant. Once this rate is known for the specimen A, assumed as reference, the correction of the elastic prediction is obtained by multiplying the ordinates of the linear diagram by the rate between the actual secant modulus and the reference one. Clearly, the failure stress must be known to find the secant modulus. When the actual rupture stress $\bar{\sigma}_R$ is not known, but only an expected value, subsequent iterations are necessary to update this value till the last secant modulus used for the correction corresponds to the expected $\bar{\sigma}_R$ in the $\sigma(\varepsilon)$ curve.

The tests made on the specimens E with the smaller thickness have pointed out no evident influence of the thickness on the results.

Specimens F, geometrically similar to the A ones but smaller, show the scale effect, typical of fracture mechanics; nevertheless this effect is lower than the linear elastic prediction because of the strong material non linearity. Specimens A and B, which have very different notch depths but reach the same value of K for the same $\bar{\sigma}$, present instead no evident differences of $\bar{\sigma}_R$. This can be explained by considering that the influence of the material non-linearities increases with σ and practically depends on the nominal value $\bar{\sigma}$.

Conclusions

The results of the tensile tests made on cast iron notched specimens agree with the predictions of the field criteria. Nevertheless, considering the material used, the linear elastic predictions are not quantitatively reliable and some corrections are necessary to take into account the non-linearities of the grey cast iron. The proposed correction does not have a general validity and is justifiable only for geometries and load conditions closed to the tests ones. To obtain a larger experimental basis it would be desirable to carry out further tests and to make non-linear numerical techniques ready. Commercial finite element codes cannot be used for this case, because the implemented constitutive laws do not usually agree with the grey cast iron behaviour.

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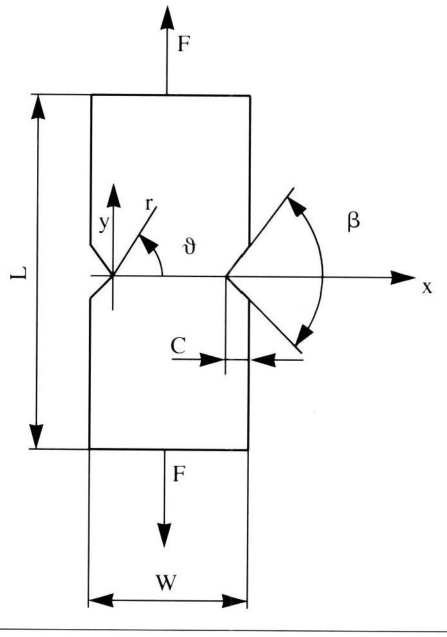

Fig. 1:
Specimen geometry.

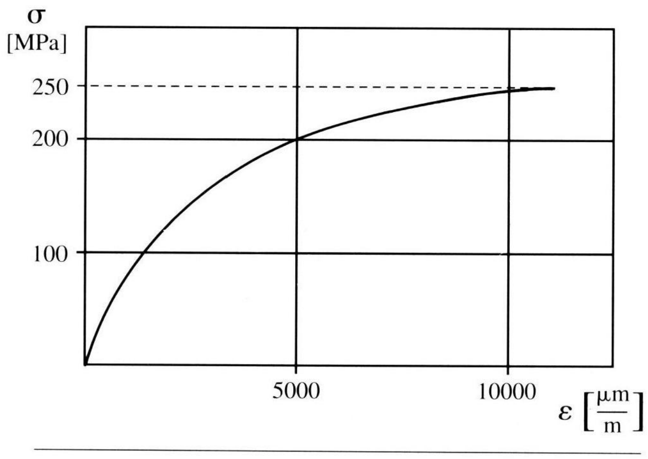

Fig. 2:
Experimental σ(ε) curve.

Fig. 3:
Metallurgical structure (200x, Nital 3%).

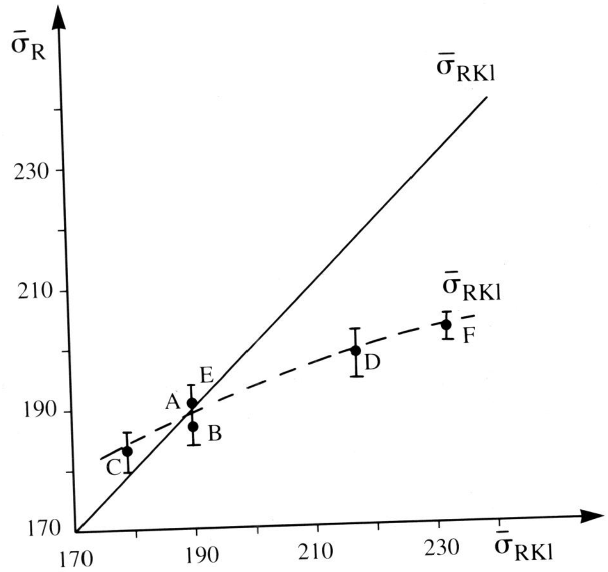

Fig. 4:
Comparison between the test results and the theoretical predictions.

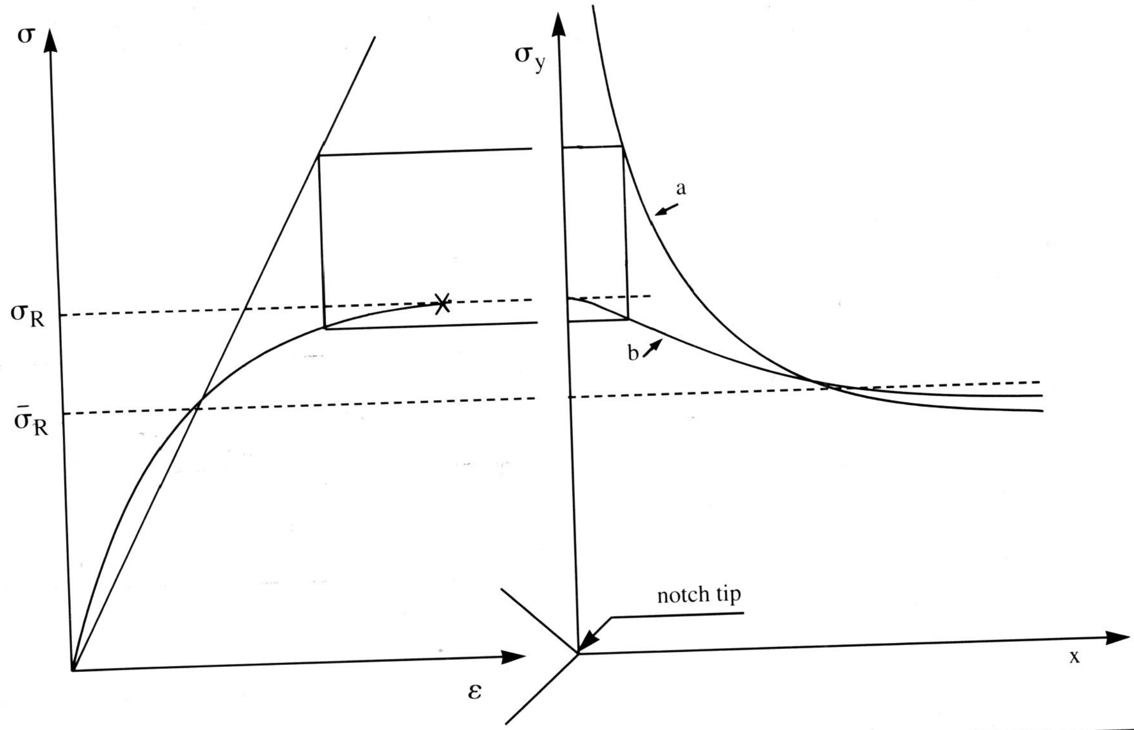

Fig. 5:
Experimental $\sigma(\varepsilon)$ curve and σ_y stress distribution (near the notch) versus x axis: linear prediction (a) and non-linear corrected diagram (b).