A Preliminary Study on Fracture Mechanics Tests in the Presence of V-Notches

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Abstract
The stress singularity due to the presence of V-notches is discussed, and the basic conditions for the theoretical applicability of field criteria for faylure analysis are examined. The present study is aimed to give a geometric definition of specimens undergoing tensile stress.

Riassunto
Si considera la singolarità del campo tensionale dovuta ad intagli a V e si valutano le condizioni da soddisfare perché sia teoricamente applicabile un criterio di campo ai fini della verifica di resistenza. Il presente studio è finalizzato alla definizione geometrica dei provini da sottoporre a trazione.

Introduction

The failure analysis of machine components with stress concentration, is generally carried out with reference to the nominal stress and introducing the stress concentration factor into the allowable strength calculation. Such a coefficient, apart from a rather unpracticable case-by-case experimental determination, can be derived from the theoretical stress concentration factor and notch sensitivity. Such a procedure, however, is hardly applicable in case of sharp angle notches because of the stress field singularity. In such circumstances, an indetermination area is likely to emerge as the theoretical notch coefficient would be infinite and notch sensitivity would tend to zero.

The aim of the research under course is to fill up such a gap, and in particular to investigate whether criteria similar to the ones employed in the fracture mechanics will still apply. In other words, the question is to ascertain if and when the fracture can be correlated to the typical quantities of the stress field singularity obtained by the elastic solution.

The present study falls within the above research program and is specifically aimed to define the most suitable shape to be conferred to the specimens, as well as the required accuracy of machining operations. In particular, the various disturbance causes likely to affect the stress field are discussed, and the extension of the disturbed areas as well as their effects on the field criteria applicability are assessed.

Singular stress fields

With reference to plane problems and a V-notch with stress free surfaces, the analytical expression of the singular integral of the homogeneous equations is well known in the case of unlimited domain [3], [5]. Such a singularity referring to the angle bisector, may consist of two components: a symmetric component and an antisymmetric one. Assuming a polar reference with origin at the notch tip (Fig. 1) and considering the symmetric portion, the singular stress field is represented by the following equations:

$$\sigma_x^* = \frac{K}{r^\alpha} \cdot \{(2+A)\cos(\alpha\theta) + \alpha \cos[(\alpha+2)\theta]\} \tag{1}$$

$$\sigma_y^* = \frac{K}{r^\alpha} \cdot \{(2-A)\cos(\alpha\theta) - \alpha \cos[(\alpha+2)\theta]\} \tag{2}$$

$$\tau_{xy}^* = \tau_{yx}^* = \frac{K}{r^\alpha} \cdot \{A \sin(\alpha\theta) + \alpha \sin[(\alpha+2)\theta]\} \tag{3}$$

Having placed

$$A = \cos[2(1-\alpha)\theta_o] + (1-\alpha)\cos(2\theta_o):$$

For the antisymmetric part, instead, we will have:

$$\sigma_x'^* = \frac{K'}{r^{\alpha'}} \cdot \{-[2+A')\sin(\alpha'\theta) + \alpha' \sin[(\alpha'+2)\theta]\} \tag{4}$$

$$\sigma_x'^* = \frac{K'}{r^{\alpha'}} \cdot \{-[2-A')\sin(\alpha'\theta) - \alpha' \sin[(\alpha'+2)\theta]\} \tag{5}$$

$$\tau_{xy}'^* = \tau_{yx}'^* = \frac{K'}{r^{\alpha'}} \cdot \{A' \cos(\alpha'\theta) - \alpha' \ \cos[(\alpha'+2\theta]\}, \tag{6}$$

having placed

$$A' = \cos[2(1-\alpha')\theta_o] - (1-\alpha')\cos(2\theta_o),$$

where the exponents α and α' are a function of the notch opening angle β and $\theta_o = \pi - \beta/2$. These exponents are the solution of the transcendental equations

$$\frac{1}{1-\alpha} \sin[2(1+\alpha)\theta_o] + \sin(2\theta_o) = 0, \tag{7}$$

for the symmetric field, and

$$\frac{1}{1-\alpha} \sin[2(1-\alpha)\theta_o] - \sin(2\theta_o) = 0, \tag{8}$$

for the antisymmetric field. The exponents trend in terms of β is shown in Fig. 2. It is worth noting that the antisymmetric singularity is weaker than the symmetric one, and only occurs if $\beta < \sim 102°$. The symmetric one is always present provided the corner is re-entrant ($\beta < 180°$).

Using an extension of the Fracture Mechanics words, the constants K and K′ will be referred to as generalized stress intensity factors.

Actually the stress field deviates from the pure theoretical singularity due to various reasons:

— boundary conditions not equivalent to the asymptotic ones of the singular field:
— effects of geometric non-linearity due to unlimited strains at the notch tip;
— microstructural non-homogeneity of material;
— geometrical manufacturing inaccuracies of the tip;
— material non-linearity.

Conditions for field criteria applicability to the failure analysis of notched parts

In case of finite domain without volume forces, stresses can be thought of as summation of a singular part expressed by eqs. (1, 6) and a non-singular part necessary to meet the boundary conditions, i.e.:

$$\sigma_{ij} = \sigma_{ij}^* + \Delta\sigma_{ij}. \tag{9}$$

The non-singular part $\Delta\sigma_{ij}$ depends on boundary conditions. In relative value such a term, which is important near the boundary portion other than the notch sides, vanishes when getting near the notch tip itself. Assuming that the fracture starts from the notch tip and the actually important stress field is the immediately surrounding one, a zone in the neighbourhood in which the disturbance $\Delta\sigma_{ij}$ is practically negligible.

The amplitude of such a arca depends on specimen size and geometry, in addition to the load application mode. One can thus recognize two areas shown in Fig. 3 with A and B; the eq. (9) prevails in the area B, whilst in the area A one can deem that σ_{ij} (x, y) $\simeq$ σ^*_{ij}(x, y), can be defined.

The non-linearity and microstructural lack of homogneity of materials cause deviations of the actual behaviour from the linear-elastic solution.

In fact, the stress field of the analytical solution does not match with the reality at the notch tip, as at a microscopic scale one can no longer consider the material structure as homogeneous or the linear solution as accurate because of the large strains taking place there. Moreover, structural materials generally deviate to a certain extent from Hooke's law prior to fracture. These effects are at a peak close to the notch tip while they decrease getting far from it. Finally the inaccurate accomplishment of a sharp notch (due to defects in machining operations) also involves some changes in the stress field. Actually, without a sharp corner, the singularity foreseen by theory would disappear.

With reference to a given material, just prior to fracture, considering a domain in Ω_ω and assuming that the material presents an initial linear portion, we can in principle recognize the areas A_0, A_1, A_2, A_3 shown in Fig. 4. In the area A_0 the theoretical computation is not meaningful due to lack of homogeneity of the material; the area A_1, usually inclusive of A_0, is the site of non-linearities; in A_2, instead, one can appreciate the disturbances induced by areas A_0 and A_1 versus the theoretical linear solution; in the area A_3, the linear-elastic solution is practically valid. When considering a notched element (and thus a limited domain), the linear elastic fracture mechanics (LEFM) is surely applicable if the area A_2 is within the area A, previously defined. Such a situation is reported in Fig. 4. If the area A limit is included within the area A_2, it may happen that the boundary disturbance, adding to the ones due to non-linearities, changes the extension of the area A_1. In this case, one shall apply the non-linear fracture mechanics (LEFM). Indeed, the LEFM will apply if the disturbance coming from the boundary affects the area A_1 (Fig. 5). Eventually, if the boundary disturbances come to affect A_o, the field criteria will no longer apply.

Effect of the boundary proximity

The disturbance induced by the boundary proximity was quantified for a load condition involving the presence of the only symmetric part of the singular field. By a boundary elements code [4], [8] generally suited to problems related to fracture mechanics [7], the stress fields were analysed in the case of rectangular section specimens, with two symmetric notches having angle $\beta = 90°$ under axial load (Fig. 6). Comparing the pure singular field σ^*_{ij} to the specimen one, two boundaries were assessed within which the differences $\Delta\sigma_{ij}$ (9) are not over 5% and 10%, respectively, of the maximum principal stress. Fig. 7 shows the extensions of such boundaries as a function of the ratio (W-2C)/W, where C indicates the notch depth and W the specimen width. For small notch depths, there prevails the disturbance due to the finite lenght of the corner sides, whereas for higher depths the disturbance due to the proximity of the opposite boundary will prevail. Such considerations justify the fact that there is a depth C for which the extension of practically undisturbed area is at a maximum. In particular, for the specimens under survey this occurs for a value of the ratio (W-2C)/W estimated around 0.5, this value being considered optimal.

Fig. 8 shows the shape factor $\varnothing_y$ related to the ratio (W-2C/W)

$$\varnothing_y = \frac{K(2 - A - \alpha)}{C^{\alpha} \cdot \sigma_{nom}}$$

where σ_{nom} indicates the ratio between the applied force F and the net resistant section, while the other quantities are the ones already referred to this factor enables to find K, to be inserted into the formulas (1, 3), for opening angle $\beta = 90°$ for any notch depth.

Similar investigations were carried out on specimens with different geometries and load conditions without however obtaining — the specimen width being equal — larger areas with small disturbance.

Effect of geometric defects due to machining operations

The geometry of defects, as such, is not widely known. The magnitude order of the ratio between the defect extension and the disturbed area extension can be studied considering for simplicity sake a circular fillet radius at the notch bottom end. The graphs in Figs. 9 and 10 report the stress trend on the bisector in direction of the axis x and y, in the cases of pure singularity and blended recess. One can note that differences vanish when getting far from the recess bottom end, so that a distance can be located beyond which differences become negligible. The stress computation was performed, by the above mentioned boundary elements code, imposing the pure singular displacements ou a sufficiently distant boundary.

In particular, it was found out that for distances equal to 0.97 and 1.91 times the fillet radius, the disturbance for σ_y is about 5% and 10%, respectively.

When there is a fillet radius r the stress field is no more singular and the theoretical stress concentration factore α_k can be related to the generalized stress intensity factor K by the equation:

$$\alpha_k \ (r) \cdot \sigma_{nom} = \lambda \frac{K}{r^{\alpha}} \tag{10}$$

where K and α are the ones already defined and λ is a proportionality factor that does not depend on the radius provided this is sufficientely small.

Equation (10) can justify that, under conditions of similitude, maximum stress criteria are applicable using the notch intensity factor

$$\beta_k = 1 + \eta_k \ (\alpha_k \ (r) - 1)$$

where r is a small and arbitrary radius and the notch sensivity is an adjusted function of r.

Effect due to the material inelastic behaviour

This type of disturbance was assessed as excess approximate order by the method described earlier. The yielding of the material was schematized assuming as limit hypothesis, that small portion at the singularity does not contribute to withstand internal stress. This is the same as removing the material within a boundary of the tip which for simplicity is assumed to be circular. In order to evaluate the influence of the form of such boundary, the case of a crack with a depth equal to the diameter of the area previously considered was also analysed. In Figs. 11 and 12 a comparison is made between the pure singularity and the stress fields arising in the cases of material removal within a circle with unit radius and a crack with unit depth. In both cases the disturbance disappears when getting far from the damaged area. After material removal, at distances of 5.7 and 9.2 radii, one can see differences of 5% and 10% respectively, while in the presence of a crack such differences will respectively appear at a distance of 4.7 and 7.6 times the length of the crack itself. This is a further confirmation that the extension of the disturbed area is not much effected by the way of yielding of the material, on condition that it is localized.

Conclusions

As regards the execution of experimental tests aimed to determine the fracture strength of V-notched materials, it was defined the optimized depth to be ascribed to the specimen notch undergoing axial load. Further, the conditions to be met were evaluated in order to get significant results from tests both in linear and non-linear fields.

Waiting for test results, we can in principle assume that fillet radii no higher than the dimensions of structural non-homogeneities are uninfluential. Moreover for materials with a markedly plastic behaviour, we can suppose that even larger fillet radii are uninfluential, provided that they are small in comparision with the plasticized area.

References

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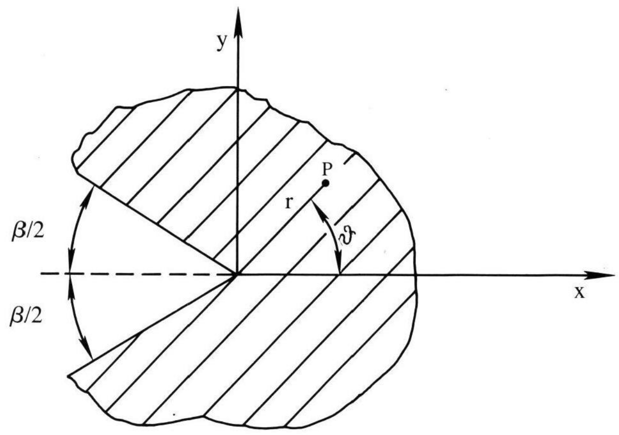

Fig. 1:
Reference system at the notch tip.

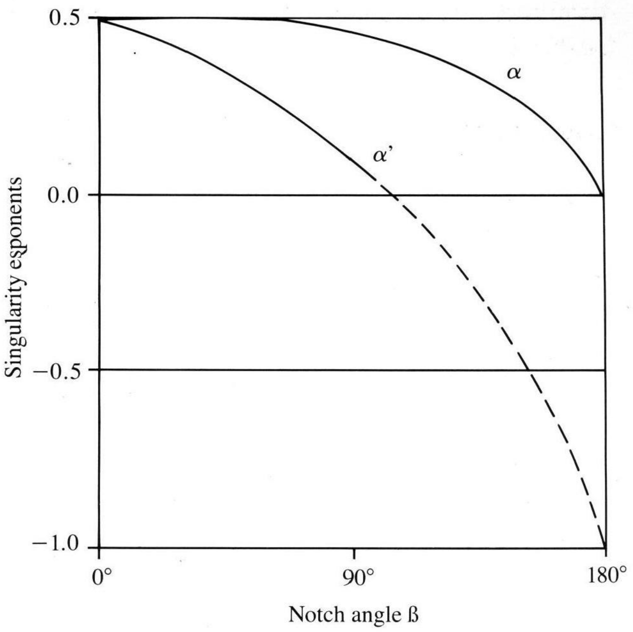

Fig. 2:
Singularity exponents α, α′.

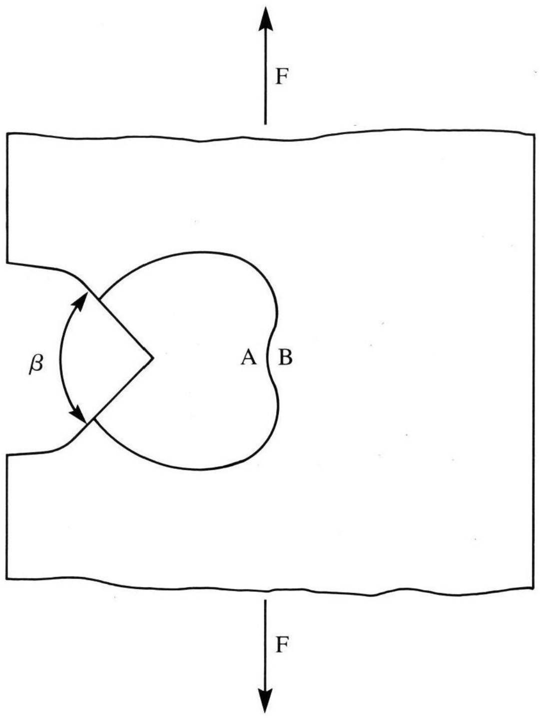

Fig. 3:
Definition of the areas A and B within which the elastic-linear solution is not affected, (A) and it is influenced (B) by the boundary effect.

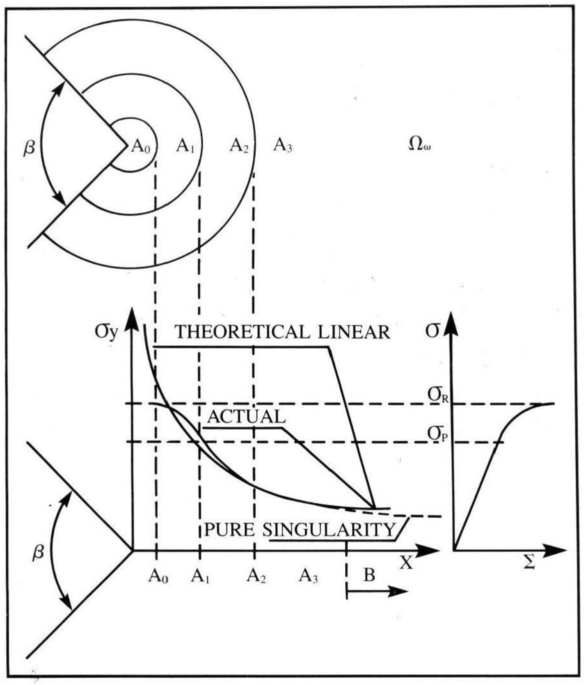

Fig. 4:
Trend of the actual stresses and singular solution under applicability conditions of the elastic-linear fracture mechanics.

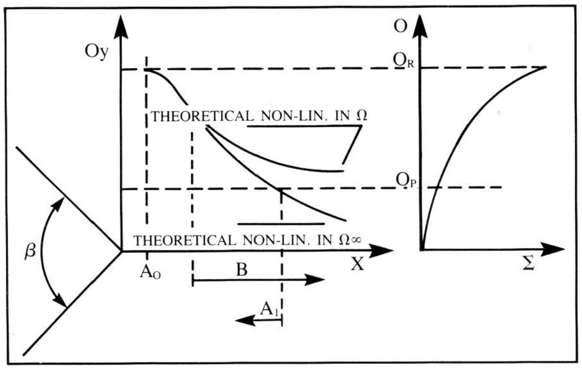

Fig. 5:
Trend of the actual stresses, process curve and associated singular solution under applicability conditions of the non-linear fracture mechanics.

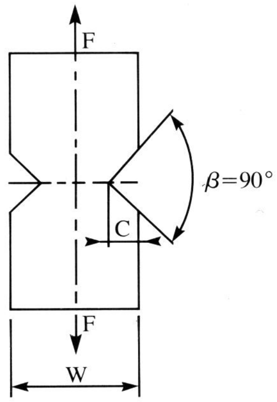

Fig. 6:
Notched specimen.

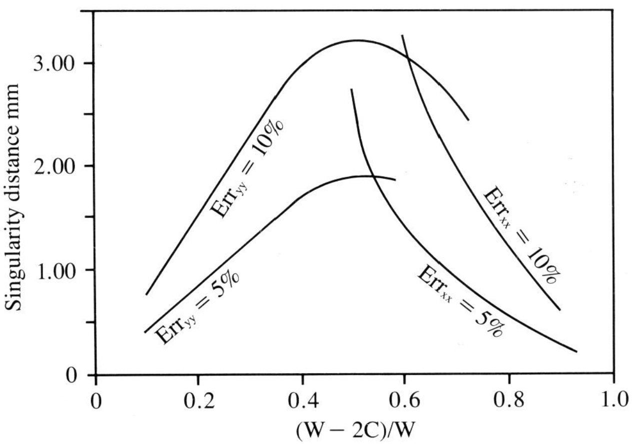

Fig. 7:
Extension of the areas within which the deviation from the singular solution amounts to 5% and 10%.

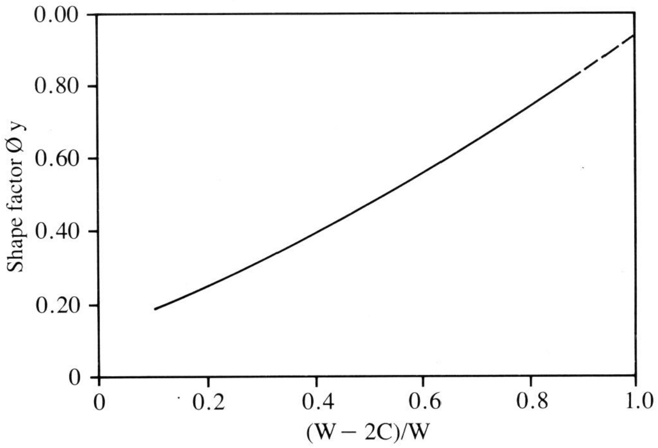

Fig. 8:
Shape factor $\varnothing_y$.

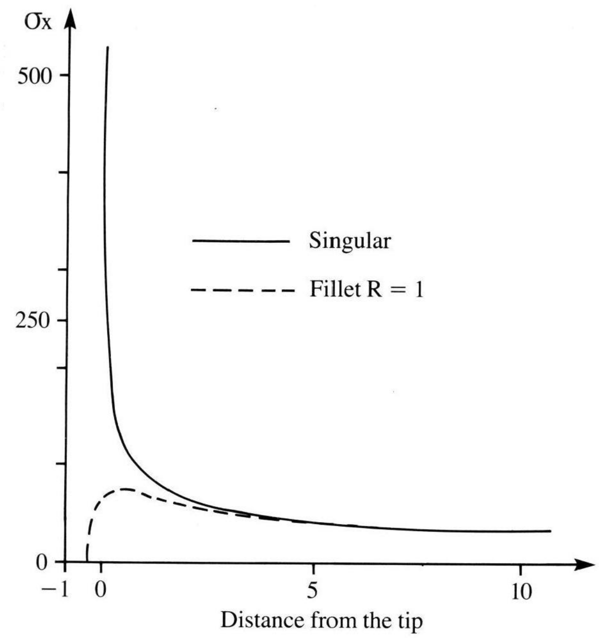

Fig. 9:
Comparison among σ_x in cases of pure singularity and unit radius fillet along the notch bisector.

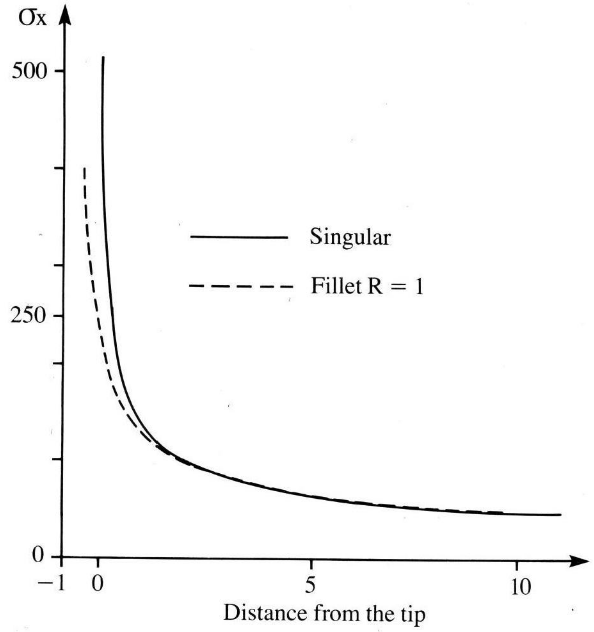

Fig. 10:
Comparison among σ_y in cases of pure singularity and unit radius fillet along the notch bisector.

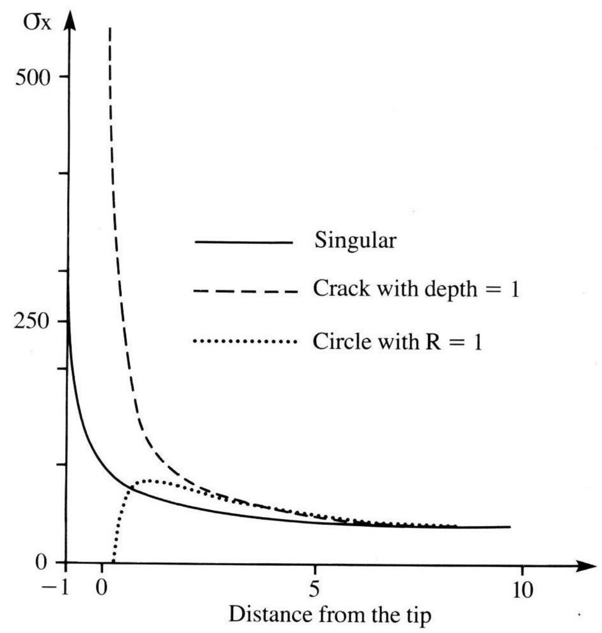

Fig. 11:
Comparison among σ_x in cases of pure singularity, circle of unit radius and unitary depth along the bisector.

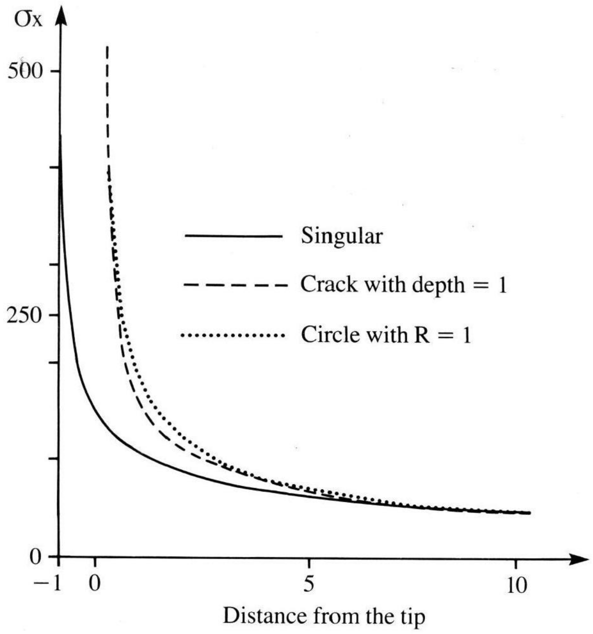

Fig. 12:
Comparison among σ_y in cases of pure singularity, circle of unit radius and unitary depth along the bisector.