A computer program for the calculation of stresses and strains in the axisymmetric cup drawing

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Abstract
The proposed computer program is developed incorporating previous results by WOO (1), as well as taking into account the cup wall, whose exact solution had not been previously sought. The calculations are performed in this case starting both from the punch side as well as from the die side, so as to individuate the position of the section of the cup wall on both sides of which the computed values of stresses and strains are the same within defined limits. The results are in this way generalized to cup walls that form contact angles different from 90° with the initial blank position.
KEY WORDS: Cup drawing, Deep drawing, Cup wall.

Riassunto
Determinazione al calcolatore di tensioni e deformazione nell'imbutitura assi-simmetrica
Per la preparazione di un programma di calcolo di tensioni e deformazioni nell'imbutitura assialsimmetrica, gli autori, partendo da una memoria di WOO (1), tengono conto anche della parete dell'imbuto, prima non considerata. I calcoli sono eseguiti iniziando sia dal centro del pezzo che dal bordo e procedono in modo da individuare la posizione di quella particolare sezione della parete, a monte ed a valle della quale i valori di tensione calcolati sono gli stessi entro un'approssimazione prestabilita. I risultati sono poi generalizzati a pareti che formano angoli anche diversi da 90° rispetto alla posizione dello sviluppo piano.

Setting out the problem

Stresses and strain analysis in an axial symmetric cup drawing process of a blank has been tackled both by " compact " computation and by finite elements method (1), (2), (3), (4), (5), (6) or by finite difference method (7).
The present work has originated from a critical reading of a paper written by D.M. WOO (1). It aims at providing a simple, effective and comprehensive compact computation methodology of stresses and strains in the process of cup drawing without resorting to previous assumption on the blank-die or blank-punch contact angles.

Computation analysis

The paper develops a complete compact solution of an axial symmetric cup drawing according to WOO's theory, being given the geometry of the die and punch, the punch diameter, the thickness and the physical characteristics of the sheet material, the drawing ratio and the blank holder force F; as a result it provides the stresses and the strains in the drawn blank and the force P to be applied to the punch (Fig. 1).

The blank has been divided, as it has been made by WOO, in six parts of a predetermined geometry (Fig. 2).

A is the region with constant thickness under the blank holder, *B* the region with a decreasing thickness up to the beginning of the die lip, *C* the region with decreasing thickness on the die lip. Then *X* refers to the region with constant thickness under the punch, *Y* to the curved region with decreasing thickness under the

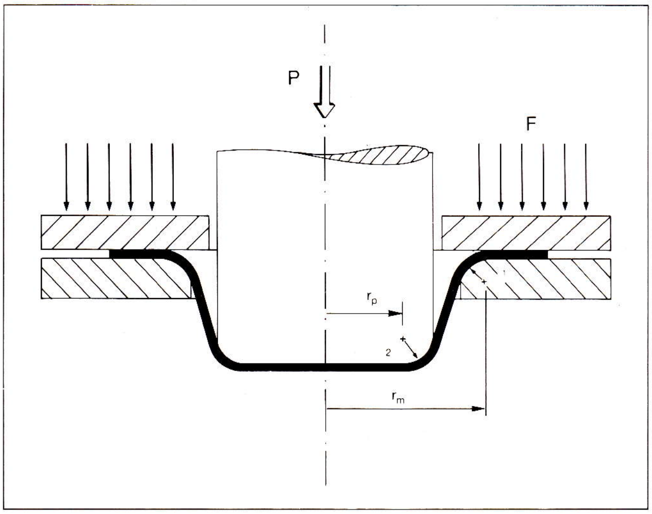


Fig. 1 - Axisymmetric cup drawing process.

Fig. 2 - Stress and strain computation regions.

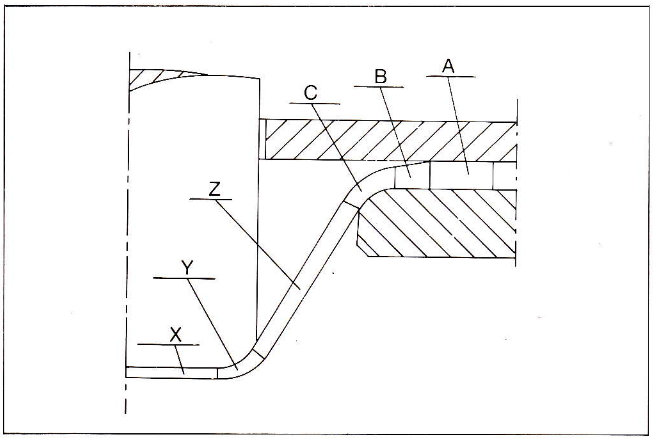


Fig. 3 - Definition of the contact angles ϑ.

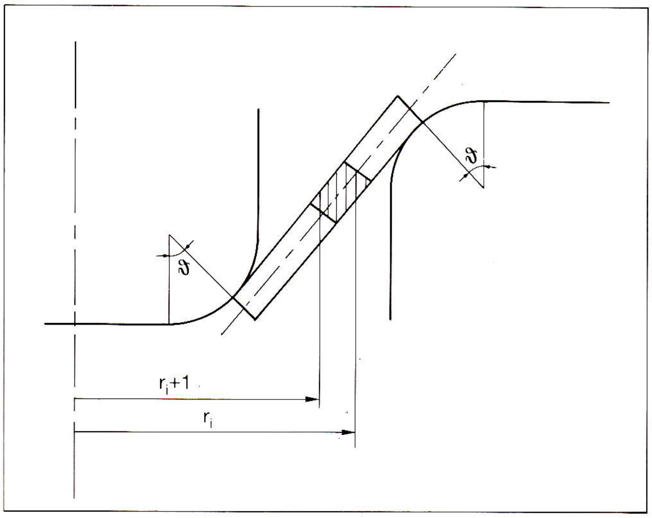
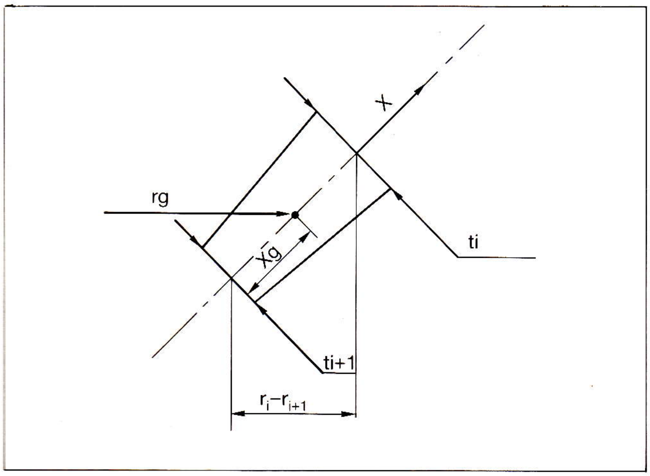


Fig. 4 - Symbols as used for the cup wall region.



punch and *Z* to the one between the die and the punch.

Z is assumed to be rectilinear, with its sides tangent to the fillets of the punch and of the die and with decreasing thickness from the die to the punch. The angles ϑ of contact between the blank and the die profile and between the blank and the punch profile are therefore assumed as equal to each other (Fig. 3).

The vertical forces exchanged between the die and the punch are also equal to each other and to the force P applied to the punch. This force is a function of ϑ angles, but is also implicitly determined by the above indicated drawing ratio; therefore a check is introduced in the program between the volume developed from the drawn blank for a general angle of contact ϑ and the corresponding volume of unstrained material. The exact value of ϑ and therefore of the force P are thus generated.

The volume of one element in the Z region has been calculated as follows (Fig. 3 and 4):

$$V = S \cdot 2\pi \cdot r_G \tag{1}$$

where S is the area of the longitudinal section and r_G the radial coordinate of the baricentre.

By indicating as r_i, r_{i+1} the limit radii of the element and as t_i, t_{i+1} the corresponding thicknesses, one obtains for a general contact angle ϑ:

$$S = (t_i + t_{i+1})/2 \cdot (r_i - r_{i+1})/\cos\vartheta; \tag{2}$$

$$r_G = r_i - x_G \cdot \cos\vartheta \tag{3}$$

where

$$x_G \approx (r_i - r_{i+1})/2 \cdot \cos\vartheta \tag{4}$$

The volume of the element, in the unstrained blank of thickness t_0, between the radii R_i and R_{i+1} (capital letters are used throughout for the undeformed state) is

$$V = (R_i^2 - R_{i+1}^2) \cdot t_0 \cdot \pi \tag{5}$$

From the law of constancy of the volume the following relationship can be derived:

$$\begin{aligned}&(R_i^2 - R_{i+1}^2) \cdot t_0 = \\ &= (t_i + t_{i+1})\,(r_i - r_{i+1})\,[r_{i+1} - (r_i - r_{i+1})/2]/\cos\vartheta\end{aligned} \tag{6}$$

In the above mentioned paper of WOO (1), the control of the constancy of volume is not considered, as in his computation procedure, in a very approximate way, an angle of contact of 90° is assumed.

Structure of the program

The blank has been divided in N elements as concentric circular crowns, whose extremes have been numbered from the external to the internal by subscripts i and i + 1.
The cup drawing process has been divided in M steps j, where j = 1 represents the unstrained structure.
Herebelow the input data of the program are shown:

ρ die profile radius

r_m radius to the die lip

ρ_2 punch profile radius

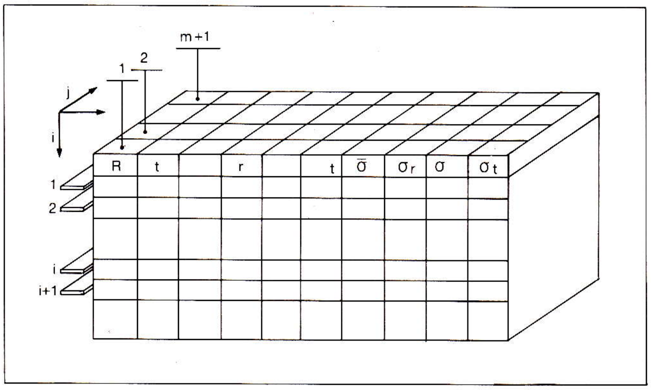


Fig. 5 - Three dimensional representation of the initial and calculated variables values.

r_p radius of the flat base of the punch

μ coefficient of friction

F blankholder force

$\bar{\sigma}_0$ initial yield stress

A, n constants in the stress-strain relation ($\bar{\sigma} = A\bar{\varphi}^n$)

$\bar{\sigma}_{max}$ max. von Mises yield stress

N number of elements

M number of steps

$\delta\ (r/R)_B$ incremental parameter of drawing ratio: $(r/R)_B = 1 - \delta\ (r/R)_B$; (r/R_B) is the drawing ratio; r_B is the drawn blank radius

R_B initial blank radius

t_0 initial thickness of the blank

δt_1 thickness ratio of *A*; $\delta t_1\ t_0 = \delta t_A$

δt_2 thickness ratio of *X*; $\delta t_2\ t_0 = \delta t_X$

For each step the output is a table; for each value of i the following variables are reported (Fig. 5):

R initial radius of a particular element in the blank

t current thickness

ϑ angle of contact between the blank and the die profile equal to that of the blank and the punch profile

r current radius

φ_ϑ principal circumferential strain

φ_t principal thickness strain

$\bar{\sigma}$ von Mises yield stress

σ_r principal radial stress

σ_ϑ principal circumferential stress

σ_t principal thickness stress

The computation starts from the region *A* and, through *B* and *C*, reaches the region *Z*; then it starts again from the region *X* and, through the region *Y* reaches once

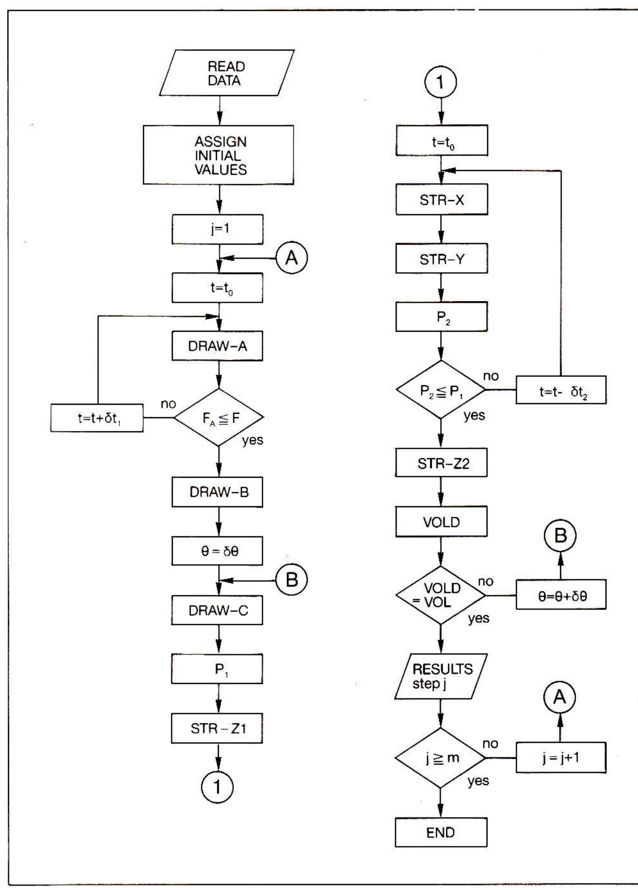


Fig. 6 - Program flow chart.

more the region *Z*. Thus the computation in the region *Z* is performed twice; the boundary i, of the element of *Z*, whose stresses σ_r and σ_t, obtained by two different approaches, are as much as possible equal, is the boundary point looked for. In order to determinate it, it has been adopted a criterium by which the deviation of σ_r and σ_t

$$| (\sigma_r)_{z1} - (\sigma_r)_{z2} | / | (\sigma_r)_{z1} + (\sigma_r)_{z2} | \qquad (7a)$$

$$| (\sigma_\vartheta)_{z1} - (\sigma_\vartheta)_{z2} | / | (\sigma_\vartheta)_{z1} + (\sigma_\vartheta)_{z2} | \qquad (7b)$$

are at a minimum.

Generally a boundary point for σ_r and a different boundary point for σ are determined; to discriminate between them the point where the deviation of the other variable is lower is chosen.
The computation is carried out by means of subroutines named in the same way as the regions to which they refer (DRAW-A for the region *A*, DRAW-B for region *B*, DRAW-C for region *C*, STR-X for region *X* and STR-Y for region *Y*).

The subroutine indicated as STR-Z1 computes stresses and strains in the region *Z* starting from *C*, whereas STR-Z2 is the suobroutine which carries out the same computation starting from region *Y*. Fig. 6 reports the computer program flow sheet.

The subroutines are articulated in a loop j corresponding to the generic step of the cup drawing process and controlled by the parameter M, the following loops being nested within:

— a loop t for the computation of the thickness of the region *A*, controlled by the value of the force F of the blankholder;

— a loop t for the computation of the thickness of the blank in the region *X*, controlled by the coincidence between the forces P_1 and P_2 of the punch, computed in the regions *C* and *Y* respectively, for a general angle ϑ;

— a loop ϑ for the computation of the contact angle in the regions *C* and *Y*, controlled by the coincidence between the volume of the drawn blank, VOLD, and the volume of the unstrained blank, VOL.

The volume VOLD is of course equal to the sum of the volumes of the regions *A, B, C, X* and *Y* and of the region Z1 up to the extreme i of the boundary element, and of the region Z2 up to the same extreme.
The depth h of the drawing piece is computed as follows:

$$h = (r_e - r_f) \cdot tg\vartheta \qquad (8)$$

where

$$r_e = r_m - \rho_1;\ r_f = r_p + \rho_2 \qquad (9)$$

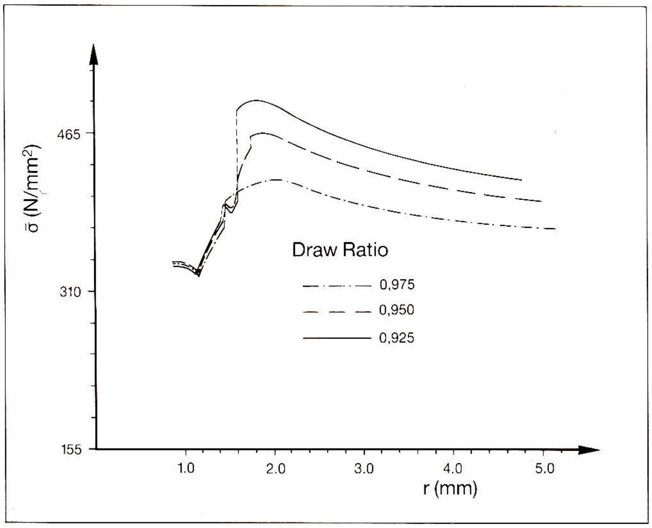


Fig. 7 - v. Mises yield stress $\bar{\sigma}$ vs. current radius r as calculated for three different drawing ratios.

Results and applications

In the diagram in fig. 7 the von Mises yield stress $\bar{\sigma}$ as a function of radius r and for three drawing steps j is shown. It is derived from the computation with the input data shown in Table 1. These data are the same as those used in one of the examples in W00's paper (1).
The results agree perfectly with those obtained by W00 for the regions *A, B* and *C*, while no comparison is possible with region *Z*, not calculated by W00, and for the regions *X* and *Y* which have been computed by that author with different input data.
In Table 2 the percent error of volume, the drawing depth and the corresponding force applied to the punch for three different drawing ratios are shown.
The algorithm for the complete solution of the problem is written out in a Fortran program of about 1200 statements available upon request.
The executable module requires about 32 KB. RAM and therefore it can be easily implemented on a microcomputer.
The authors have tested the program on three computers of different classes:
— Host computer Olivetti-Hitachi 5560 (TSO environment);
— Workstation Apollo DN 3000 (4 MB. RAM);
— IBM PC-AT (5 12 KB. RAM, with mathematical coprocessor Intel 80287).
Table 3 compares the different execution times.

TABLE 1 - Input data as given by WOO (1), and transformed in metric units

ρ_1	$=0.51$ mm	$\bar{\sigma}_{max}$	$=570.5$ N/mm^2
r_m	$=3.17$ mm	N	$=90$
ρ_2	$=0.51$ mm	M	$=3$
r_p	$=2.03$ mm	$\delta(r/R)_B$	$=0.04$
μ	$=0.13$	R_B	$=5.33$ mm
F	$=3924$ N	t_O	$=0.089$ mm
$\bar{\sigma}_O$	$=317.7$ N/mm^2	δt_1	$=0.02$
A	$=446.4$ N/mm^2	δt_2	$=0.002$
n	$=0.575$		

TABLE 2 - Data in output for different drawing ratios

Drawing ratio	0.975	0.950	0.925
% Error of volume	0.780	0.850	4.913
Drawing depth (inch)	0.11	0.26	0.27
Punch force (ton)	29.685	45.910	48.196

TABLE 3 - Comparison of the performances of different computer systems

	Time		
Computer	CPU	I/O	Total
Host c. Olivetti-Hitachi 5560 (TSO environment)	9″	1′ 29″	1′ 38″
Apollo Workstation DN 3000 (4 MB Ram)		≈5′	
IBM PC-AT (512 KB Ram) mat. copr. INT. 80287		≈11′	

Conclusions

A readily available computer program to calculate stress, strains and punch loads in various steps of a cup drawing process has been developed.
The mathematical computation has as start point a procedure proposed by W00 (1); this procedure takes into account contact angles between the cup wall and the blank or the punch base of 90°. By an hypothesis of a rectilinear cup section the procedure has been extended to contact angles different from 90°.
The program considers 90 different points along the blank radius.
This simple model seems to fit the situations which are present during the cup drawing; its relatively low computing time offers an advantage in its use.

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