TWO LEVEL FATIGUE LOADING (H-L) OF MG ALLOYS: MICROMECHANIC MODELING VS. EXPERIMENTS

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Abstract

A statistical micromechanic fatigue model shows some of the main features of macro fatigue behavior like S-N power law, endurance limit, Goodman diagram, etc. The model correlates micro statistical damage parameters to the macro behavior. The recursive evolution equation has been recently transformed to a nonlinear differential form, which enables a simple analytic solution. Using these results, a two level (H-L) fatigue loading case is modeled analytically, leading to a failure design formula which is based solely on the one level S-N data. Specifically, the model predicts a generalized "Miner type" behavior, which is controlled on the micro-level by the very basic strength dispersion factor of the microelements, and on the macro-level by the S-N power (slope). The proposed H-L predictions were tested by fatigue experiments on two Magnesium alloys (AZ31 and AM50). Results showed good correlation, in spite of the natural large scatter.

Riassunto

Un modello statistico di fatica a livello micromeccanico rivela alcune caratteristiche del comportamento macroscopico a fatica, come la legge esponenziale S-N, il limite di fatica, il diagramma di Goodman, ecc. Il modello correla il danneggiamento statistico a livello microstrutturale col comportamento macro. L'equazione di evoluzione ricorrente è stata recentemente trasformata in una forma differenziale non lineare, che permette di arrivare ad una semplice soluzione analitica.Usando questi risultati, una situazione di carico affaticante a due livelli (H-L) viene modellata analiticamente e porta ad una formula di progettazione a durata basata unicamente sui dati S-N ad un solo livello. Specificatamente, il modello predice un comportamento "alla Miner" generalizzato, che è controllato a livello microstrutturale dal fattore di dispersione della resistenza dei microelementi, e a livello macro dall'esponente della curva S-N (pendenza). Le previsioni H-L fornite dal modello sono state state verificate sperimentalmente tramite prove di fatica su due leghe di Magnesio (AZ31 e AM50). I risultati hanno mostrato una buona correlazione,a dispetto del naturale elevato livello di dispersione.

Cordiali saluti

INTRODUCTION

The use of microstructure parameters in constitutive equations for damage evolution and failure prediction is under extensive study (examples: Mahesh et al., 1999; Devillers et al, 1997). Microscale approaches are especially important for modeling fatigue life, since the major source of failure is related to small, non-uniform properties such as moduli, strength, residual stresses etc. Recent multiscale models (Mura and Nakasone, 1990; Papadopoulus, 1999) use types of maximum stress criteria on specific slip planes, to define fatigue limit. Differential approach to fatigue damage, including microstructure parameters has been studied also (Fedelich, 1998). One of the first microscale approaches was to consider bundles of elements having statistically dispersed properties and examine their collective behavior. This model, originally proposed for textiles (Daniels, 1945), is continuously developing, especially for fiber composites (Phoenix and Beyerlein, 2000). Many microscale models have been proposed on damage mechanics (McCartney and Smith, 1983; Krajcinovic and Van Mier, 1999), using various kinds of element properties (Zweben and Rosen, 1970; Vujosevic and Kracjinovic, 1997; Mahesh, Beyerlein and Phoenix, 1999). However, studies on fatigue failure are still in demand.

The specific problem of two level (H-L) fatigue loading is very old (Miner, 1945; Manson et. al., 1967). It is the first multilevel loading step towards random (realistic) protocols and shows a severe deviation (for design purposes) from the classical linear cumulative (Miner) prediction.

In spite of the enormous complexities on the microscale, all materials exhibit a common and simple macro failure response (S-N, da/dN-ΔK). Thus, it is plausible to model fatigue behavior by neglecting microstructure details, and consider only micro properties that are necessary and are common to almost all materials.

A simple micromechanic fatigue model, based on the above "bundle approach" has been developed in previous studies (Altus, 1991; 1995) with realistic macro response such as the S-N power law, endurance limit and Goodman diagram. This model is extended here to receive an explicit H-L formula.

BASICS OF A MICROMECHANIC FATIGUE MODEL – SINGLE LOAD LEVEL

Consider a simple model composed of an ensemble of 1D linear elastic (until break) elements. The array is organized as a parallel set and loaded uniformly by a strain ε. All elements have a stiffness E and a statistical strength distribution $F(\varepsilon)$, such that

$$F(\varepsilon) = \int_0^{\varepsilon} f(\varepsilon')d\varepsilon' \qquad (1)$$

and $f(\varepsilon')$ is the probability density function (pdf). This simplified model can be visualized as an idealized part of a polymer, where the elements represent molecular chains, or a unidirectional fiber composite, where the elements represent fibers, or even metals, where grains are the basic constituents. Motivated by the fact that all materials exhibit similar "macro failure appearance" (S-N, da/dN-ΔK curves etc), microstructure details are neglected (enormous complexity).

Let a cyclic (fatigue) loading be applied such that the macro stress ranges between $\sigma_{min}=0$ and $\sigma_{max}=\sigma_0$, i.e., R=0. During the first loading, weak elements break and the remaining part sustains the outer load. From 1-D equilibrium equation one gets:

$$\sigma = E_0 \cdot \varepsilon_1 \cdot (1 - F(\varepsilon_1)) \qquad (2)$$

where E_0 and ε_1 are the elements stiffness and the maximum strain of the first loading, respectively. E_0 is also the initial modulus of the ensemble. $F(\varepsilon_1)$ elements (relative number) break during the first loading. Eq. (2) can be also written as:

$$E_1 = \frac{\sigma_1}{\varepsilon_1}, \quad F(\varepsilon_1) = B_1 \qquad (3)$$

or (using(2)):

$$1\text{-}F_1 = E_1 \qquad (4)$$

where B_m is the damage which occurs during cycle m and F_m is a short writing for $F(\varepsilon_m)$.

In real materials, local stress concentrations near broken elements may lead to additional breaks at the same loading cycle, but from simplicity reasons this possibility is neglected here. However, the effect of broken elements on their non-broken neighbors is taken into account in a different way. During the consequent loading cycle, the local microstructure in the vicinity of each broken element undergoes many types of relative movements due to irreversible motions: friction, dynamic effects, local heating, bifurcation states etc. Without entering to a detailed analysis, the end result of the local reorientation, between the first stress peak and the second can be separated into two categories. In the first, the local neighbor strain is *higher* than the one for a non neighbor element (strain concentration higher than 1) and in the second it is *lower*. It is the first type, which is fundamentally important for fatigue, since it is the main cause for cyclic damage.

Each "interacted" neighbor of the first type has a different stress (or strain) concentration factor, according to the specific local micro-geometry. From a statistical point of view, only two parameters are needed for further analysis: the relative number of neighbors of the first type and the average of this local stress (strain) concentration factor. In this study, the analysis is simplified even further, by assuming a locally (near neighbor) very high strain concentration, which practically causes *definite* failure during the consequent cycle. It is also assumed that the reason for broken-neighbor interaction depends on the initial local geometry only, i.e., a non-interacting neighbor will not re-interact during subsequent cycles. However, this type of neighbor can still fail as a non-neighbor element when experiencing a strain, which is higher than all previous values.

Returning to the second cycle, $F(\varepsilon_1)$ "old" breaks cause their neighbors to fail during the second loading. In addition, new isolated breaks also appear, since the strain which is needed for the same stress, is higher for the second cycle. Therefore, it is necessary to divide the damage during the second cycle into two parts: Coming from expansion of old breaks, and creating new sites (index a and b, respectively). Then,

$$B_{2a} = F(\varepsilon_1) = 1 - E_1 \equiv F_1 \qquad (5)$$

$$B_{2b} = \frac{F_2 - F_1}{1 - F_1} \cdot \left[1 - F_1 - B_{2a}\right] = \frac{2E_1 - 1}{E_1}\left[E_1 - (1 - F_2)\right] \qquad (6)$$

$$B_2 = B_{2a} + B_{2b} = E_1 + (1 - F_2)\left[\frac{1}{E_1} - 2\right] \qquad (7)$$

where (4) has been used. Now apply equilibrium for the second cycle:

$$\sigma = (1 - B_1 - B_2) \cdot \varepsilon_2 \quad ; \quad E_0 = 1 \qquad (8)$$

or, using (3,4) and (7):

$$1 - F_2 = \frac{E_1 E_2}{2E_1 - 1} \qquad (9)$$

The third cycle is calculated similarly:

$$B_{3a} = B_2 \qquad (10)$$

$$B_{3b} = \frac{F_3 - F_2}{1 - F_2}(1 - B_1 - B_2 - B_{3a}) \quad (11)$$

Using equilibrium as above, and rearranging we get:

$$1 - F_3 = \frac{E_1 E_2 E_3}{(2E_1 - 1)(2E_2 - E_1)} \quad (12)$$

Now, from (4),(9) and (12) it is easy to predict the solution for any cycle n:

$$1 - F_n = E_n \prod_{k=1}^{n-1} \frac{E_k}{(2E_k - E_{k-1})} \qquad n=1,2,\ldots \quad (13)$$

Or:

$$\frac{1 - F_n}{1 - F_{n-1}} = \frac{E_n}{2E_{n-1} - E_{n-2}} \quad (14)$$

Eq.(14) is a nonlinear implicit difference equation for cycle n, when the information for the two previous cycles is given. It is valid for any given strength distribution function F.

To have a specific fatigue response, a Weibull function is chosen for the statistical strength of individual elements:

$$F(\varepsilon) = 1 - \exp\left(-\frac{1}{\beta}\left(\frac{\varepsilon}{\varepsilon_s}\right)^{\beta}\right) \quad (15)$$

For the current parallel array, ε_s (statistically called "scale factor") is the strain associated with the maximum stress, which can be submitted under monotonic (static) loading conditions. It was recently shown (Altus, 2001) that by inserting (15) in (14) and converting the difference equation into a continuous form, (14) can be transformed to a nonlinear differential equation of the form:

$$E_{,kk} - \left(1 - \frac{1}{\beta}\right)E^{(-\beta)} \cdot E_{,k} = 0 \quad ; \quad k = n/N \quad (16)$$

where N is the number of cycles to failure under uniform cyclic stress conditions. The conversion is valid when the damage in each cycle is small enough, i.e., N>>1, which is the common fatigue case. Moreover, (17) has an analytic solution of the form:

$$E(n) = N^{-1/\beta}(N - n)^{1/\beta} \quad \text{or} \quad E(k) = (1 - k)^{1/\beta} \quad (17)$$

It is seen that the normalized damage evolution equation is invariant of stress. The S-N power law has been also received:

$$\frac{\sigma}{\sigma_s} \approx (1 + \frac{1}{\beta}) \cdot N^{-1/\beta} \quad \rightarrow \quad \sigma = C \cdot N^{-1/\beta} \quad (18)$$

where σ_s is the static (macro) failure stress. The relation between σ_s and ε_s is:

$$\sigma_s = E_0 \cdot \exp\left(-\frac{1}{\beta}\right) \cdot \varepsilon_s \quad (19)$$

The common design formula is $\sigma = CN^{\alpha}$, where C and a are considered as two independent material parameters. Practically, it is difficult to test if C is related to β as proposed in (18), although a positive correlation with (18) was found elsewhere (Altus, 1995). This is since σ_s is associated with fatigue failure after a single cycle, where plastic processes, which are not taken here into account here, are dominant. Fortunately, C cancels out in the final H-L results.

The differential equations (16) is of the second order, and needs two initial conditions (E(n) and $E_{,n}$ at some n) for a complete solution. Having these, an important relation is given by:

$$-\frac{E(n)}{E_{,n}} = \beta \frac{N^{-1/\beta}(N - n)^{1/\beta}}{N^{-1/\beta}(N - n)^{1/\beta - 1}} = \beta(N - n) \quad (20)$$

Thus, (20) gives a simple explicit expression (which will be used later) to the remaining fatigue life, when E and $E_{,n}$ are given.

The above micromechanic model relates the macro "slope" of the classical "Wohler diagram" to the statistical shape parameter (β) on the microscale. For example, very large β values mean that the microelements are uniform (no strength dispersion), which, on the macro response yields a minimal fatigue effect. Practically, many materials exhibit the above power law (linear relation on a log(S)-log(ν) plot and $5<\beta<20$ is found for most materials. Further generalization of the model are given elsewhere (Altus, 1995, Altus and Jeulin, 2000).

The study in the following aims to explore the fatigue life predictions for the two levels High-Low case. The engineering importance of this loading sequence stems from the fact that the fatigue life is usually much lower than the simple "Miner law" predictions. While many studies have been focused on this problem, the models are still phenomenological (Hashin and Rotem, 1978; Hashin, 1980; Manson and Halford, 1986).

Two simplifying assumptions of the model in its present form are: a.) At each cycle, all new cracks are "evenly spread", and old ones do not coalesce, and b.) The crack "size effect" is neglected, i.e., the cyclic crack growth rate is equal for all microcracks. These approximations are valid as long as the major damage progression comes from many small microcracks, rather than from a single macrocrack. Since the major portion of fatigue life is spent before a single dominant crack is observed, model predictions can be still useful.

HIGH-LOW TWO LEVEL LOADING

Consider the case where the specimen is loaded first n_H cycles at a stress level $\sigma_{max}=\sigma_H$ ($\sigma_{min}=0$) and the rest at $\sigma_{max}=\sigma_L$ until failure. Damage evolution of the different stages of the process is illustrated schematically in figs.1.

The H part (until point A in fig.1) is calculated directly from the analytic solution of the one level loading (17):

$$E_A = E(n=n_A) = N_H^{-1/\beta}(N_H - n_H)^{1/\beta} \quad ;$$

$$\varepsilon_H = \frac{\sigma_H}{E_H} \quad ; \quad n_H = n_A \qquad (21)$$

where N_H is the one level (high) fatigue life and E_A is the stiffness at n_A. When changing from σ_H to σ_L (From A to A' in fig.1), the maximum cyclic strain decreases by a ratio equal to σ_H/σ_L. Since the non-neighbor (new cracks, i.e. B_{ib}) breaks appear only when experiencing a strain higher than all previous strains, there is an intermediate stage (from A to B in fig 1), when only the old cracks are expanding, and there are no new crack formations.

Therefore, the damage rate of this stage is constant and equals to $E_{,n}(n_H)$, i.e.,

$$E_{,n}(n_H) = -\frac{1}{\beta} N_H^{-1/\beta}(N_H - n_H)^{(1/\beta)-1} \qquad (22)$$

which remains constant as long as the maximum cyclic strain is lower than the largest strain experienced in the whole loading history. At point B, The strain reaches its value from A again, and a third regime of damage evolution starts. Then, using (20) we have

$$\frac{E_B}{E_A} = \frac{\sigma_L}{\sigma_H} = \left(\frac{N_L}{N_H}\right)^{-1/\beta} \quad . \qquad (23)$$

Therefore, the number of cycles at the second stage is:

$$(n_B - n_A) = \frac{E_B - E_A}{E_{,n}\big|_A} = \beta(N_H - n_H)\left(1 - \frac{\sigma_L}{\sigma_H}\right) \qquad (24)$$

When the cyclic strain reaches ε_A again (point B in fig. 1), the cyclic process continues in a one level form, associated with σ_L, erasing the damage "history". Therefore, the residual fatigue life can be directly calculated by (20):

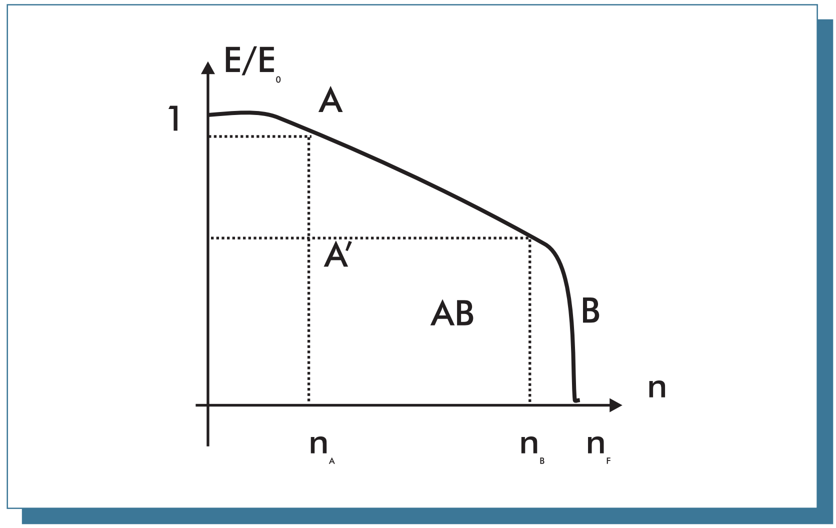

Fig. 1: Stiffness evolution of H-L loading

$$-\frac{E_B}{E_{,n}(B)} = -\frac{E_B}{E_{,n}(A)} = \beta(N_L - n_B) \quad . \qquad (25)$$

Using (22-25) we obtain a simple expression for the third stage:

$$(N_L - n_B) = \frac{\sigma_L}{\sigma_H}(N_H - n_H) \quad . \qquad (26)$$

The fatigue life of the above three stages are added to give:

$$(n_H + n_L) = n_H + \beta(N_H - n_H)\left(1 - \frac{\sigma_L}{\sigma_H}\right) + \frac{\sigma_L}{\sigma_H}(N_H - n_H) \, . \qquad (27)$$

From (18):

$$\frac{\sigma_L}{\sigma_H} = \left(\frac{N_L}{N_H}\right)^{-1/\beta} \quad . \qquad (28)$$

Inserting (28) in (27) and rearranging, we finally obtain:

$$\left(\frac{n_H}{N_H}\right) + \frac{1}{\lambda}\cdot\left(\frac{n_L}{N_L}\right) = 1 \quad . \qquad (29)$$

where

$$\lambda = (\sigma_{L/H})^{\beta}\left[\beta - (\beta-1)\cdot\sigma_{L/H}\right] \quad ; \quad (\cdot)_{L/H} \equiv \frac{(\cdot)_L}{(\cdot)_H} \qquad (30)$$

or

$$\lambda = N_{H/L}\left|\beta - (\beta-1)\cdot(N_{H/L})^{1/\beta}\right| . \qquad (31)$$

Eqs.(29) and (31) express an explicit prediction of fatigue life under H-L loading sequence. Since $\lambda(\beta, N_{H/L})$ is independent of n_H or n_L, (29) appears as a straight line on the familiar (n_H/N_H, n_L/N_L) space, and can be considered as a generalized "Miner type" relation. Previous models (Manson and Halford, 1986) proposed a "double linear" rule, but were phenomenological. Note that (29) depends solely on the power of the S-

N Wohler diagram ($1/\beta$), and contains no additional "material parameters" to be fitted.

It is seen (30) that $0<\lambda<1$ for all materials and stress ratios. It reduces to the Miner law only in the trivial case (i.e., $\lambda=1$) when $\sigma_{L/H}=1$ (for any β) or $\beta=1$, which is not physical in our case. λ is also a monotonically decreasing function of β, which means that higher fatigue resistive materials (larger b), will show a higher sensitivity to the H-L loading (lower λ). This is consistent with experiments.

Another important prediction of (29) is that even a very small amount of H cycles can reduce the total fatigue life considerably. This will be demonstrated by the following experimental results.

EXPERIMENTAL RESULTS AND MODEL TESTING

Two Magnesium alloys were tested: AZ31 extruded (18mm extruded and machined to 10mm) from ALUBIN and AM50 rolled (3mm raw thickness, cut to 10mm width specimens) from ROTEM. S-N diagrams, shown in fig.2 for both materials were received under 6 Hz and R=0 conditions by a standard (Instron) loading machine. β values (slope = $1/\beta$ on the S-N logarithmic plane) were found to be around 7.1 for both alloys. The observed dispersion does not affect the robustness of the slope. This reference information was needed to ensure that the same conditions (size, specimens, surface characteristics, fixtures, loading frequency etc.) are used both for the one level fatigue and for the H-L loading. Two loading levels of H-L fatigue testing of AZ31 were conducted: $\sigma_H/\sigma_L=[0.8/0.65]\sigma_u$ and $[0.8/0.65]\sigma_u$. The high scatter enforced testing of many specimens, so that the H-L stress levels were chosen for a "manageable" fatigue life in the range $5*10^3$ to $7*10^4$. For further confidence, another set of AM50 specimens under $\sigma_H/\sigma_L=[0.8/0.7]\sigma_u$ conditions was tested. The large scatter calls for some normalization, which will enable accumulation of data from many materials and conditions into one plot. This can be achieved by writing Eq.(29) in a "Miner" form:

$$\frac{n_H}{N_H}+\frac{\bar{n}_L}{N_L}=1 \quad ; \quad \bar{n}_L=\frac{n_L}{\lambda} \qquad (32)$$

In this way, a plot of all results on the normalized plane can be done relative to a "generalized" Miner line. These data, collected in fig.3 for all three cases above, show the general predictive potential of the model, in spite of the large scatter.

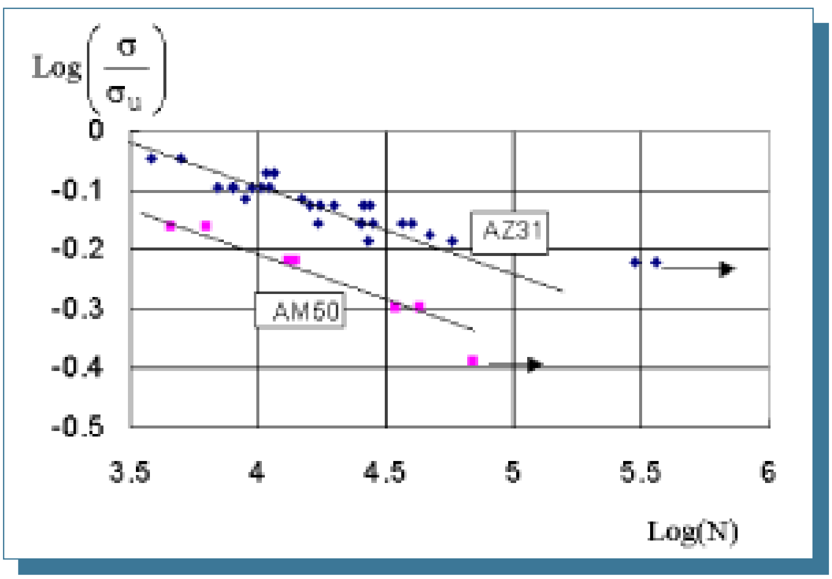

Fig. 2: S-N curves (R=0) for AM50 (rolled) and AZ31 (extruded) Mg alloys. σ_u(AM50)=265MPa, σ_u(AZ31)=276MPa. Linear numbering on logarithmic scales is used for easier b calculations

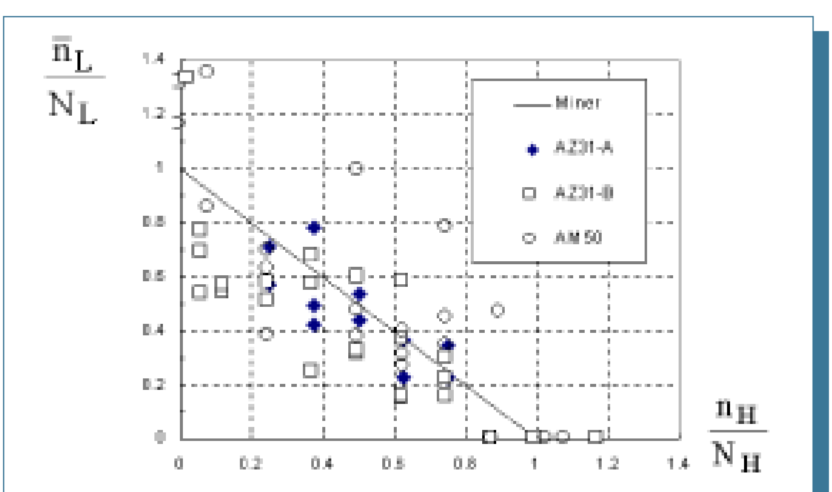

Fig. 3: Comparison between model (eq.31) and three types of experiments: AZ31-A,B are AM50

DISCUSSION AND CONCLUSIONS

Some additional insight is summarized in the following.

One

The formula (29) is simple and explicit, and therefore convenient for engineering design. Moreover, the only material parameter involved is β, which is the reciprocal of the slope of the basic logarithmic S-N curve. Since no additional testing or material constant is needed for its immediate use, it is more a predictive tool, rather than merely a "sophisticated curve fitting".

Two

The predictive capabilities come mainly from the fact that the model contains some micromechanical information, which is difficult to contain in a macro type model. This is the *first neighbor correlation*, which basically takes into account the fact that under external macro stress, material elements will have different probability of failure if their first neighbor have failed or not. Of course, higher order information (i.e., dependence on two or more neighbors) can be used, but are they necessary? For example, it is clear that a material element near a large crack (of many elements) will have higher probability of failure. However, the existence of large cracks means that the material is in a damage state very close to total failure. Therefore, the ratio of modeling effort to predictive benefits may be doubtful.

THREE

A recent fatigue study on human bones (Zioupos and Casinos, 1998) helps in testing the model predictive capability, as seen in fig.4. The high β value (taken from a separate S-N data), causes a significant deviation from the regular Miner law, in spite of the very close stress levels of the two fatigue loading parts.

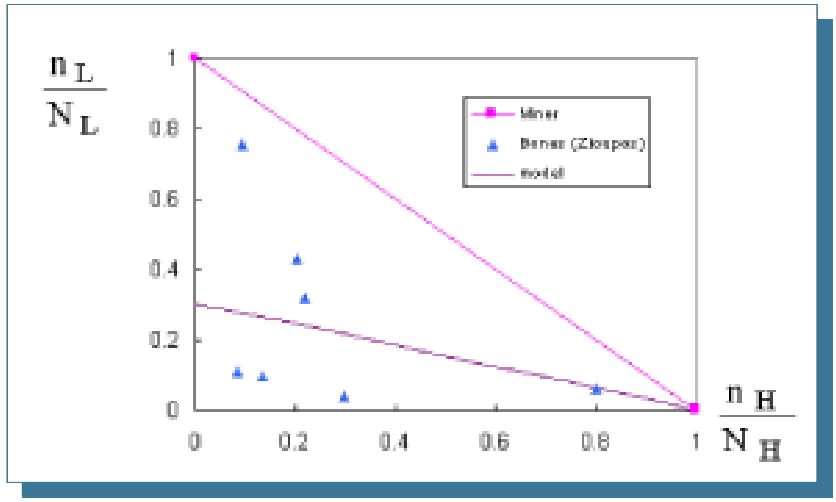

Fig. 4: H-L prediction vs experiments on human bones (β=24, σ_H/σ_L=0.9)

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