Fatigue Design of a Rough Contact

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Abstract

In the aim to build up design tools for mechanical contacts, a curve, allowing to estimate fatigue endurance of a contact between rough surfaces, is presented. The curve gives the tolerable limit value of hertzian pressure in function of the roughness of the contact bodies that is characterised by an opportune parameter. Therefore it separates the zone of infinite fatigue life from the zone of finite fatigue life for any rough contact.

The study has been developed for an elastic dry contact between a smooth plane and a rough cylinder of 100Cr6 steel in absence of friction. The fatigue damage of the contact has been valued by a multiaxial fatigue criterion that also gives the critical directions of first cracks nucleation.

The simplicity of use together with taking into account the most important phenomena in a rolling contact problem, make this curve a helpful tool in the contact design process.

Riassunto

Nell'ottica di costruire strumenti per la progettazione dei contatti meccanici, viene presentata una curva che permette di valutare la resistenza a fatica di un contatto tra superfici rugose. La curva fornisce il valore limite tollerabile di pressione hertziana in funzione della rugosità dei corpi a contatto, che viene caratterizzata attraverso un opportuno parametro. Separa perciò la zona di vita infinita a fatica dalla zona di vita a termine per un qualunque contatto rugoso.

Lo studio è stato sviluppato per un contatto elastico a secco tra un piano liscio ed un cilindro rugoso in acciaio 100Cr6 in assenza d'attrito. Il danneggiamento a fatica è stato valutato attraverso un criterio di fatica multiassiale con l'approccio di "piano critico".

La semplicità d'utilizzo, connessa con la presa in considerazione dei fenomeni più importanti in gioco in un problema di contatto rotolante, rende la curva un vero strumento d'aiuto alla progettazione di un contatto.

Introduction

Industrial mechanisms aren't formed by isolated parts but by interacting components connected together into a real mechanical system. In most of the cases the interaction is accomplished through a contact, that is often the critical point of the entire system. Nevertheless very little attention is still paid to contacts in the design phase. This negligence is due on the one hand to the traditional approaches employed in design, that bring to isolate each component dimensioning it separately, without taking into account the areas of joint and the interfaces that are indeed basics for the good working of the whole mechanism, on the other hand it can be remarked a real lack of design tools for a contact. If a new systemic approach to the mechanical design, looking to mechanisms as a whole of components interacting, could allow to solve the first problem, a lot more delicate is the issue regarding the second problem. Actually, today the Hertz [4] relations are the only available tools for a designer that wants to dimension a contact not empirically. However the reality of industrial mechanisms is pretty different from the one included into the validity domain of the hypothesis (smooth dry rolling contact, elastic isotropic homogeneous materials) of the hertzian theory developed at the end of XIX century. The approximation with the ideal hertzian case gives most of the times not realistic and doubtful results.

In the last thirty years researches in the area of contact mechanics and tribology have made lots of progresses, attaining important results; however designers haven't taken advantages from these, in witness of the academical imprint of the studies.

In the belief that in the next years designer needs have to mark out research direction in tribology, an approach oriented to design, for a contact problem rather usual in industrial mechanics, is presented in this paper. A rough contact charged with fatigue loads (e.g. contact between gear teeth or between rolling bodies and rings in a bearing) is considered, and a tool that allows to dimension it in a simple way is derived. The logical process leading to build up a design tool for such a contact entails:

- surfaces roughness characterisation by a simple parameter,
- calculation of fatigue damage for any kind of contact.

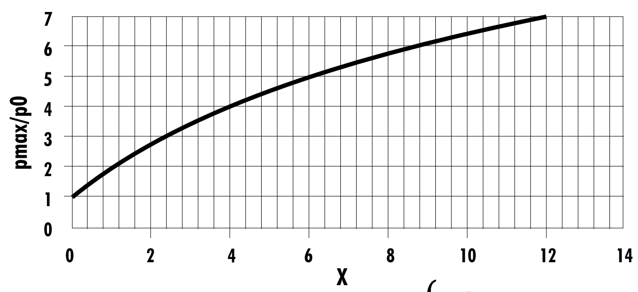
The tool provided is the *fatigue design curve* and it will be discussed in the next. The analysis has been performed for the 100Cr6 steel, a typical bearing steel, and the hypotheses considered are: 2D dry contact between a smooth plane and a rough cylinder, elastic isotropic homogeneous material, and pure rolling conditions. The cylinder roughness has been simulated by a sine curve of given wavelength and amplitude.

ROUGHNESS CHARACTERISATION

In order to estimate the incidence of roughness on the fatigue damage of a contact it is necessary to dispose of a parameter that allows to characterise the severity of surface microgeometry. The determination of the elements roughness exert its influence through, is a complex operation and, actually, roughness characterisation is the critical point of the entire approach it will be proposed in this paper.
A sinusoidal roughness, a simple model to perform analyses, is considered here. This model is not too unrealistic if it's noticed that most of machining processes leave on surface pieces undulations with a given wavelength that could be approximated by sine waves and remembering that any surface signal can be decomposed into a series of sine functions with Fourier analysis. Nevertheless real roughnesses are pretty different than simple sine waves, and the problem of their characterisation is an open question and asks for a rapid solution.
Due to the point view oriented to design needs here pursued, we aren't interested in searching the most precise and complete description, in morphological terms, of the surface microgeometry, but we want rather investigate on the influence of roughness on pressure distribution and contact stress field. Therefore it has been carried out a *functional characterisation* of roughness, that means microgeometry is characterised by considering its effects on the contact. A roughness without a considerable influence on contact stresses can be neglected, following this approach.
On the basis of these considerations, it has been chosen here to estimate roughness severity by Johnson [5] parameter:

$$X = \frac{\pi}{2}\,\frac{E'}{p_0}\,\frac{A}{\lambda}$$

where E is the equivalent elasticity modulus of the two contact bodies, p_0 the maximum of hertzian pressure, A the roughness amplitude and l its wavelength.
The parameter is proportional, among other things, to the A/λ ratio that controls, as it has been remarked by Dumont and al. [2], the overpressure of the rough contact with respect to the smooth one. Johnson has introduced the X parameter as an element to distinguish the case in which the contact area is continuous (X<1) from the one in which the contact area is discontinuous (X>1), but it's also possible to show that a relation between X and the maximal pressure of the rough contact exists (fig.1).
X allows then to characterise roughness in terms of maximal contact pressure. It takes into account microgeometry effects on the pressure field, but it doesn't consider all the consequences on the stresses: the depth where the maximum stress acts and the stressed volume don't depend on the ratio A/λ but only on the roughness wavelength λ.
It's not evident to define an amplitude and a wavelength as characteristics for any kind of roughness: beyond the approximation with a sinusoidal microgeometry, other parameters for roughness characterisation are to be searched.



$$\left\{\frac{P_{max}}{P_0} = 1 + X \qquad 0 \le x \le 1\right.$$

$$\left\{\frac{P_{max}}{P_0} = 2\sqrt{X} \qquad X > 1\right.$$

Fig.1: Curve p_{max}/p_0 in function of X

FATIGUE DAMAGE CALCULATION

The second element necessary to design a rough contact submitted to fatigue loads is a tool to evaluate damage in function of contact conditions. The approach followed for this aim has been to apply a multiaxial fatigue criterion to the contact stress field. Dang Van criterion [1] has been employed; it could be written under the form:

$$\max_h \left\{ \max_t \left[\left\| \tau a\,(h,t) \right\| + \alpha p_{hydr}\,(t) \right] \right\} \le \beta \qquad \textbf{(3)}$$

The criterion based on the observation at the microscopic scale in the material, consider responsible for the damage the alternated shear stress τ_a that acts onto the critical plane; besides the damage process seems to be facilitated by a global state of dilation characterised by a positive hydrostatic pressure p_{hyd}, or made more difficult by a global compression (negative p_{hydr}). In practice, Dang Van criterion involves the spatial research of the plane (of normal h) on which the combination of the alternated shear stress and the hydrostatic pressure is maximal, that means effecting a double maximisation with respect to h and t (time); this research can be very time consuming. The application of the criterion to simple uniaxial fatigue tests leads to determine the value of α and β constants. It has been found β=570 MPa and α=0.512 for the 100Cr6 steel we're considering here, therefore the relative weight of the hydrostatic pressure in (3) is significant.

We examine now the specific conditions of a rolling contact problem. In this case, all stresses in the bodies describe repeated compression cycles with negative mean components that, in presence of roughness, could be easily high. The orthogonal shear stress is an exception because it describes alternated cycles but that doesn't affect what it will be stated. According to Dang Van criterion, the presence of high mean compressive stresses turns out to be extremely advantageous to fatigue resistance, weakening the negative effect of the alternated shear stress. However the fatigue behaviour of a material is in reality very different. The diagrams of fig.2 helps to better understand the problem: experimental Haigh diagram and Haigh diagram led by Dang Van criterion for the 25CrMo4 steel are done. The Haigh diagram allows to evaluate the influence of mean stress on fatigue resistance. It is experimentally observed that, while a positive mean stress σ_m is always harmful for the fatigue life, in the way it lowers the tolerable amplitude σ_a of the tension-compression cycle, a negative mean stress σ_m instead, is beneficial for low absolute values of σ_m, but for higher values of σ_m a mean compression becomes again unfavourable for fatigue strength: the stress cycle amplitude σ_a decreases as it does for positive σ_m. Dang Van criterion doesn't take into account correctly this material behaviour in compression because it states that the more compression is high the more it is beneficial. That entails the not negligible risk of under-estimate the real damages, an aspect always detrimental in design.

In order to take into account mean compressions in a more realistic way and from a security side (see also Kenmeugne [6]), it has been decided here to neglect the hydrostatic pressure term, using then a criterion that keeps the same philosophy of the Dang Van one, but adapted to the specific

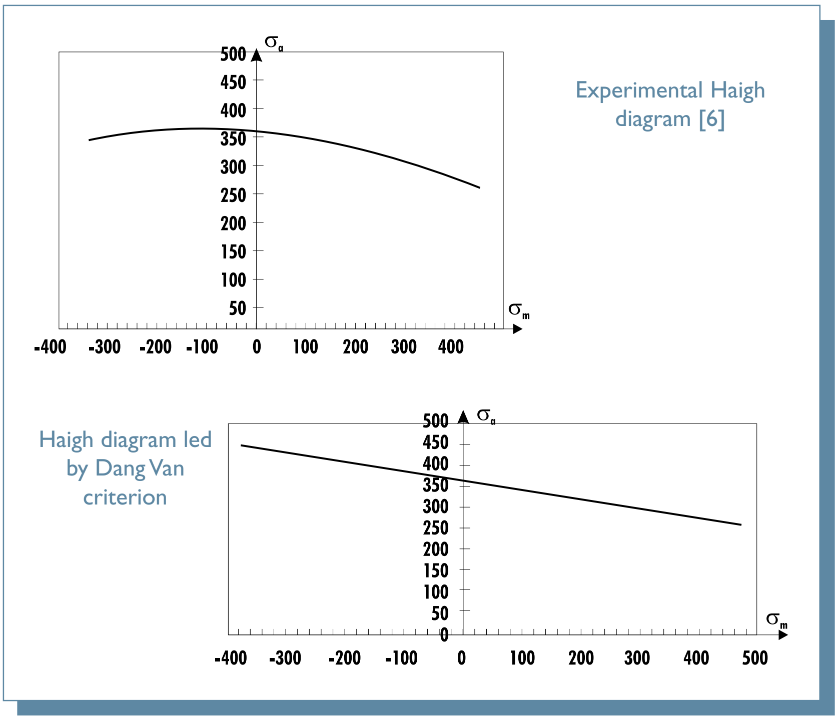


Fig.2: Haigh diagrams for the 25CrMo4 steel

case of a contact problem:

$$\max_h \left\{ \max_t \left[\left| \tau_a (h,t) \right| \right] \right\} \leq \gamma \qquad (4)$$

The constant γ is always determined with a simple uniaxial fatigue test (alternated symmetric tension) and is found equal to 355 MPa for the 100Cr6 steel.

The *criterion of the maximal alternated shear stress* doesn't take into account the mean stress effect on fatigue strength (fig.3): it is then more unrealistic in tension, but more close to reality in compression, what is of interest in a contact problem. Eventually we could imagine to use this criterion in compression and the Dang Van criterion in tension, if friction, for instance, would be present.

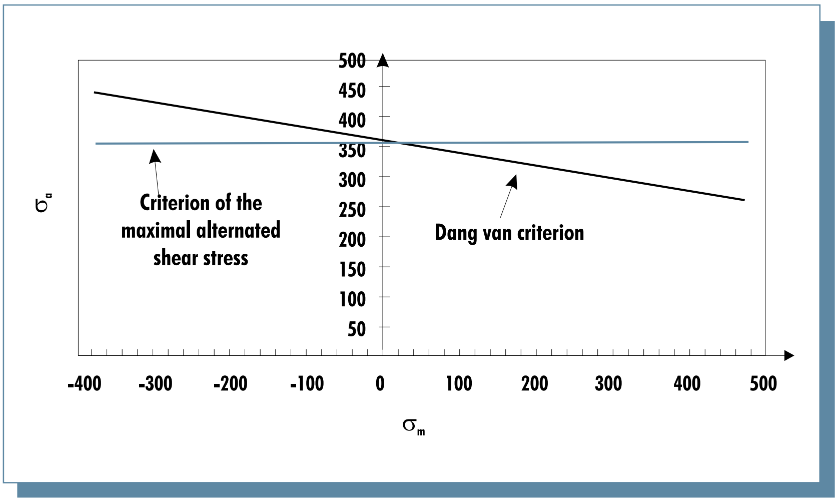


Fig.3 : Haigh diagram led by the criterion of the maximal alternated shear stress

THE DESIGN CURVE

After having sharpened the working tools, the Johnson parameter and the criterion of maximal alternated shear stress, it's now possible to obtain results than can be employed in the design phase. These results are summarized in figure 4 that represents the fatigue design curve of a rough contact for the 100Cr6 steel.

The curve is a whole of equidamaged points in the conditions of fatigue limit. It separates then the zone of infinite fatigue life from the zone of finite fatigue life for any rough contact. In function of the X parameter characterising roughness severity (in ordinates), the maximal hertzian pressure of the smooth contact equivalent, in terms of loads and geometry, to the rough one is given (in abscissa).

To provide a better understanding of the curve we shortly explain its building process. We first consider a smooth contact (X=0) and we apply the fatigue criterion to the contact stress field. The contact parameters (load and geometry) are changed until fatigue limit is reached with respect to the employed criterion. In these conditions it's possible to evaluate the maximal hertzian pressure that is indeed the tolerable limit value for the contact so that any damage occurs. We look then to a rough contact characterised by a given value of X. The same approach is followed: the fatigue criterion is applied by changing load and geometry until fatigue limit conditions are found. The application of the criterion needs calculations in every point of the contact stress field: the use of a proper software is therefore necessary. Real maximal pressure in the contact is hence known, but in the diagram we refers to maximal hertzian pressure, that means we calculate according to hertzian relations the pressure with load and geometry leading the rough contact to work at the fatigue limit. The rough contact is therefore treated as it was smooth and roughness is taken into account by the reduction of the tolerable limit value of hertzian pressure.

In other words, the problem of a rough contact submitted to fatigue loads is considered in the same way as a stress concentration problem in traditional mechanics. In that case we perform calculations as the stress raiser wasn't present and we further take it into account by the stress intensity factor; here, in the same way, we perform calculations without the consideration of roughness and we further take into account perturbations caused by surface irregularities with the design curve, by means of a reduction of the tolerable limit value of pressure.

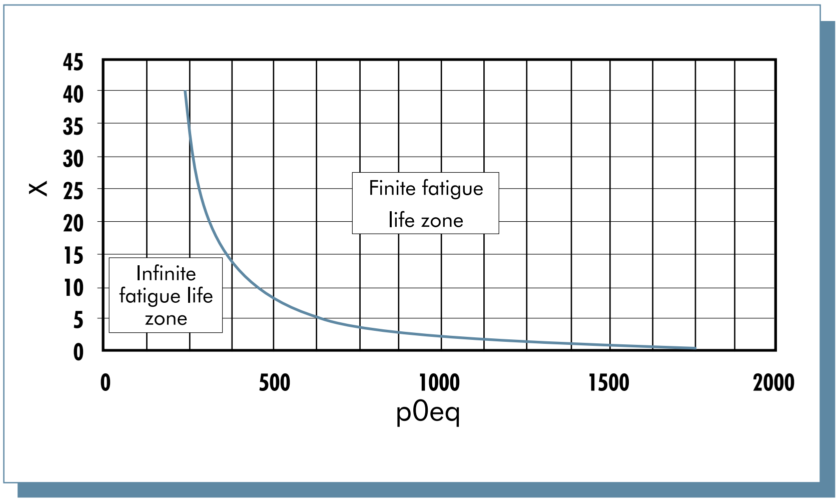


Fig.4: Fatigue design curve of a rough contact for the 100Cr6 steel

APPLICATIONS OF THE DESIGN CURVE

Some examples of application of the curve enable to better establish its properties. A real case, a contact with the following characteristics, is considered:

The limit value of the smooth contact pressure is the one derived by Lamagnère and al. [7] with a metallurgical approach, the roughness wavelength can be evaluated considering the machining process the contact bodies are submitted to. The previous ones can derive the following parameters:

- equivalent elastic modulus: $E'=E/(1-\nu^2)=230769$ MPa,
- equivalent curvature radius: $R=(R_1R_2)/(R_1+R_2)=13.04$ mm,
- maximal hertzian pressure: $p_{0eq}=[WE'/(2S\pi R)]^{0.5}=1501$ MPa.

Characteristical parameters		Value
Smooth contact limit pressure		$P_0=1770$ MPa
Roughness wavelength		l=0.2 mm
Load		W=800 N
Curvature radius of body 1		$R_1=100$ mm
Curvature radius of body 2		$R_2=15$ mm
Material:	Elastic modulus	E=210000 MPa
100Cr6 steel	Poisson coeff.	n=0.3

For the p_{0eq} found, the design curve states a maximal value of X equal to 1.12. The maximal roughness amplitude tolerable by the contact can be now derived:

$$A=\frac{2p_{0eq}\lambda X}{\pi E'}=0.93\ \mu m$$

At the end of machining processes roughness amplitude has to be lower than this value.

At the opposite, if it's known the machining processes will likely leave onto the bodies' surfaces a roughness with these characteristics:

$$A=3.2\ \mu m;\ \lambda=0.3\ mm$$

it can be verified, keeping unchanged the other contact parameters, if the contact is or not in the security domain of infinite fatigue life. The maximal hertzian pressure p_{0eq} of the contact is the same as before, because contact parameters aren't changed, and hence is equal to 1501 MPa, while the new value of X is:

$$x=\frac{\pi}{2}\frac{E'}{P_0}\frac{A}{\lambda}=2.58$$

The contact is in the finite fatigue life zone (point B of fig.5b), while the curve states a tolerable limit value of pressure of 880 MPa (point A) for the contact conditions considered. It is necessary to lower the load or modify the geometry in order to bring the contact to work in the infinite fatigue life zone. If only the load is lowered and the geometry is kept unchanged, it's necessary to go until W=100N to make the contact working in the infinite life domain. If, at the opposite, we act whether on the geometry or on the load, it can be remarked that even increasing three times curvature radius of contact bodies, load has to be lowered as well to reach conditions of infinite life. It comes out that for the following conditions:

$$R_1=300\ mm;\ R_2=45\ mm;\ R=39.13\ mm;\ W=325\ N$$

it is:

$$p_{0eq}=552\ MPa;\ X=7$$

These values correspond to a point placed along the design curve and therefore in conditions of infinite fatigue life.
The application performed shows the interest of the approach to the rough contact problem proposed in this paper. The fatigue design curve allows indeed to dimension any kind of rough contact using the simple hertzian theory. Hence it's not necessary to dispose of complex software to calculate the stress field in every point of the contact bodies, but more simply and rapidly well-known formulas of Hertz can be applied.
The curve has been provided for the 100Cr6 steel; even if we think the trend of the curve should be the same for all the steels, it is however necessary to repeat the analyses for every material in order to determine the right quantitative values.

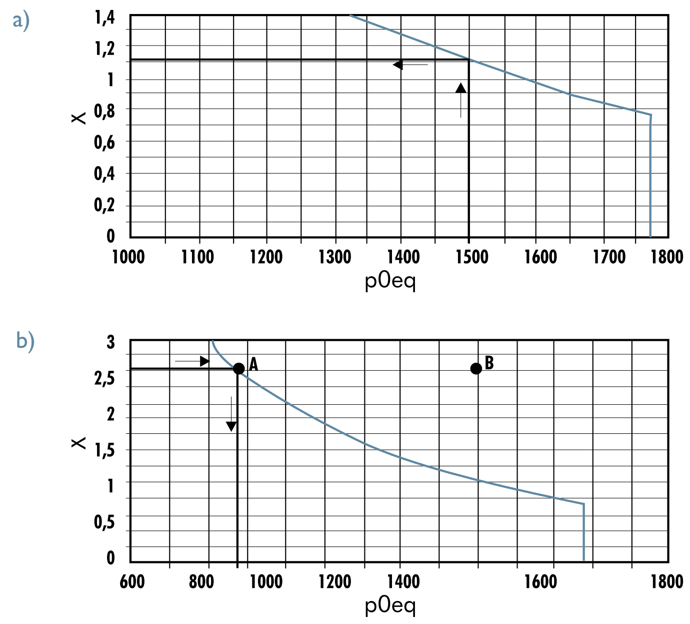


Fig.5: Applications of the design curve

The design curve has the great advantage of the simplicity of use and it's certainly a good tool to estimate the influence of roughness on contact fatigue, however it has some limits due to the same hypothesis with whom it has been built up. The curve has been established for pure rolling; the next step is to evaluate the influence of friction, even if the contacts we're considering here, in gears and in bearings, are characterised by a very low friction coefficient (≅ 0.05), that shouldn't have a great influence on the results here presented.
The hypothesis of homogeneous material has been done, while in reality, the material is not homogeneous at all, mostly in the first superficial layers, inclusions and other inhomogeneities being present. Nevertheless it has to be remarked that Lamagnère and al. [7], following a metallurgical approach based on the existence of inclusions in the material, have found in the smooth case results incredibly in accord with the ones discussed here. They obtain for the M50 steel, a bearing steel with mechanical characteristics similar to 100Cr6, a limit value of pressure for the smooth contact of 1770 MPa, while the value obtained in the same case with the approach proposed here is 1750 MPa: the result is almost the same.
Moreover the design curve has been established for a dry contact, while industrial contacts considered here are elastohydrodynamics. So it is suitable to provide a curve for EHL contacts: the results shouldn't be very different and in any case dry contact is a particular reference condition for elastohydrodynamic contacts.
In reality, however, the biggest problem of the approach to the study of a rough contact proposed here, is the roughness characterisation we've already discussed before. Another open question is the choice of variables to estimate damage. Evaluating damage only in terms of stress seems to be not complete; maybe the use of criteria that consider more variables for damage calculation (e.g. stressed volume) could be more realistic.

CHARACTERISTICS OF THE DESIGN CURVE

The design curve gives, in function of X, the tolerable limit value of hertzian pressure for the contact, that doesn't correspond, as it has already been showed, to the maximal contact pressure. The real maximum of contact pressure for each point of the curve is represented in fig.6.

Except for few fluctuations it can be noticed a constant trend. That means the criterion of maximal alternated shear stress, employed to evaluate damage, is in practice a criterion of maximal pressure. It is actually rather reasonable: due to the proportionality between stresses and pressure, to keep constant a certain value of stress is substantially equivalent to keep constant the maximum of pressure.

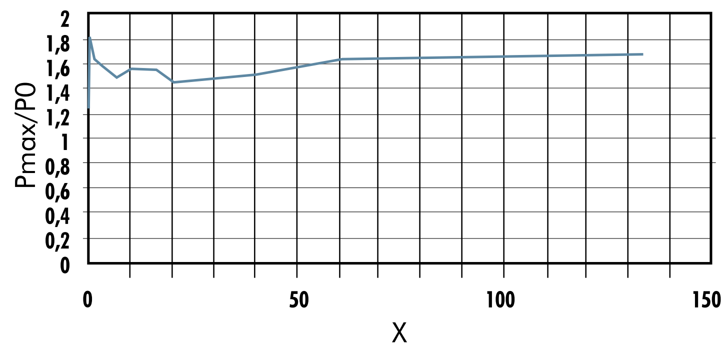


Fig.6: Maximum of contact pressure along the design curve

If we look now to the design curve, we can focus the attention on the existence of three zones of influence of roughness on fatigue damage.

For low values of X, up to about 0.75, when roughness is not very important, the damage of a rough contact is equal to the smooth one, as it's stated by the vertical part of the design curve in the zone I (fig.7a). This can be explained by considering the depth where the maximal alternated shear stress is located, that is the point of maximum damage. In a smooth contact, the maximum value of alternated shear stress is placed at the centre of the contact at a depth of 0.506a (a is half of the width of the contact area), in the hertzian zone (fig.8a). The presence of superficial roughness engenders a local increase of stresses in the bodies skin, causing a secondary maximum of damage along the depth (fig.8b). At the hertzian depth the stress field is not modified by roughness and so there is the same value of alternated shear stress as the smooth case. If roughness is very weak, the amplitude of the secondary maximum of damage in the sub-superficial zone is inferior to the value of damage in profundity, and then the critical point is located at a depth of about 0.5a, as in the smooth case. It comes out that the damage of a very few rough contact is the same of the equivalent smooth contact.

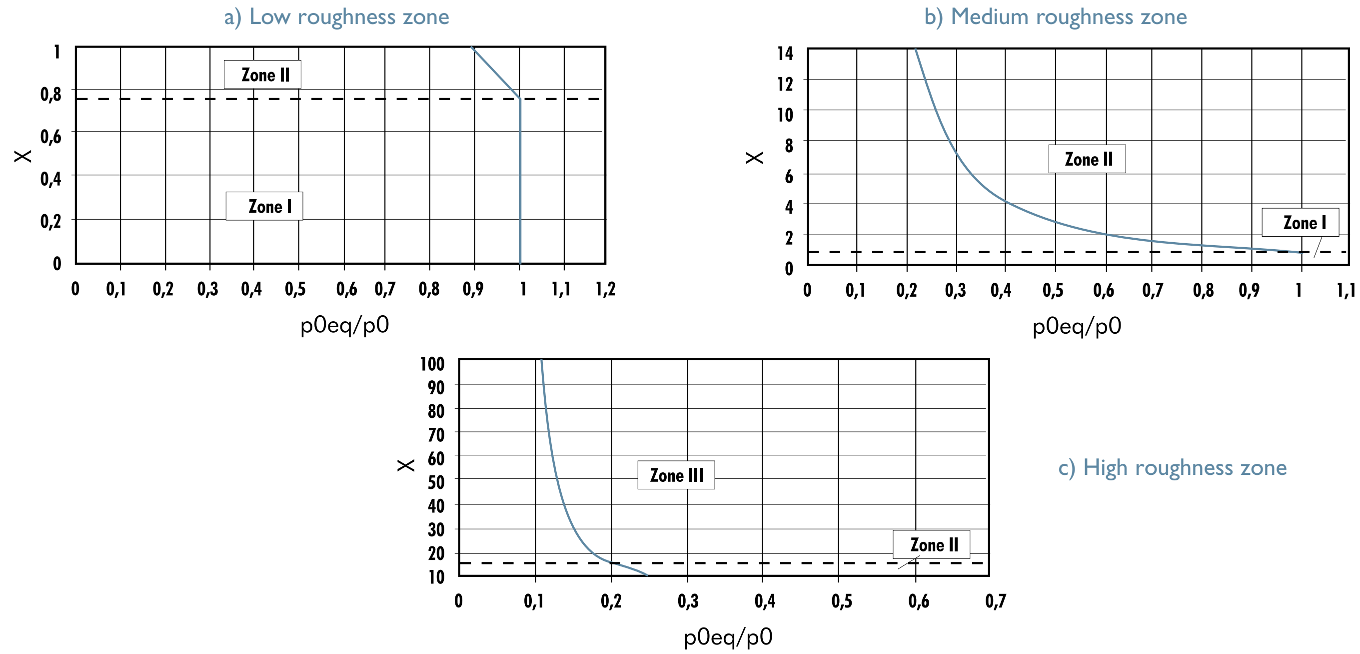


Fig.7: The three zones of roughness influence

The second zone of roughness influence is the one characterised by values of X between 1 and about 12. The roughness slope, the ratio between amplitude and wavelength that is proportional to X (1), is high enough to perturb considerably the contact stress field. As it is shown in fig.7b, a little increase of the former is sufficient to significantly lower the limit value of pressure the contact can admit. Fig.9 shows that in this case the maximum damage is in the sub-superficial zone, the one influenced by roughness, and no more in the hertzian zone, where the same damage as the smooth case is retained. Moreover it's interesting to notice that the more damaged point is not placed at the middle of the central peak of roughness but laterally shifted (fig.9b).

A third zone of influence of roughness exists (fig.7c) but it has little practical interest because it considers roughnesses very different than the ones observed in reality. In this zone a saturation effect of roughness influence is noticed: it's necessary to considerably increase the value of X in order to have an equivalent augmentation of damage, corresponding to an important decrease of tolerable limit pressure.

It can now be remarked that the fatigue criterion employed for damage calculation is a critical plane criterion, and allows then to determine in every point the plane where the maximum damage is located. The position of a plane in the space (the research of critical plane must be done in the space because the stress field is three-dimensional) is found by means of two angles:

- θ that gives the orientation with respect to surface: for q=0° the plane is parallel to the surface,
- φ that gives the orientation with respect to rolling direction: for j=90° the plane is parallel to this direction.

Critical planes for contacts in zone I and II obtained by calculations are:

ZONE I

	plane A	plane B	plane C	plane D	plane E
φ	90	90	180	180	180
θ	45	135	0	90	180

For low values of roughness, the criterion foresees cracks nucleation at 45° with respect to surface and parallel to rolling direction corresponding to the critical planes A (90°, 45°) and B (90°, 135°). This is perfectly in accord with experimental observations led by Dumont [3] on smooth specimens after contact fatigue tests in pure roll-

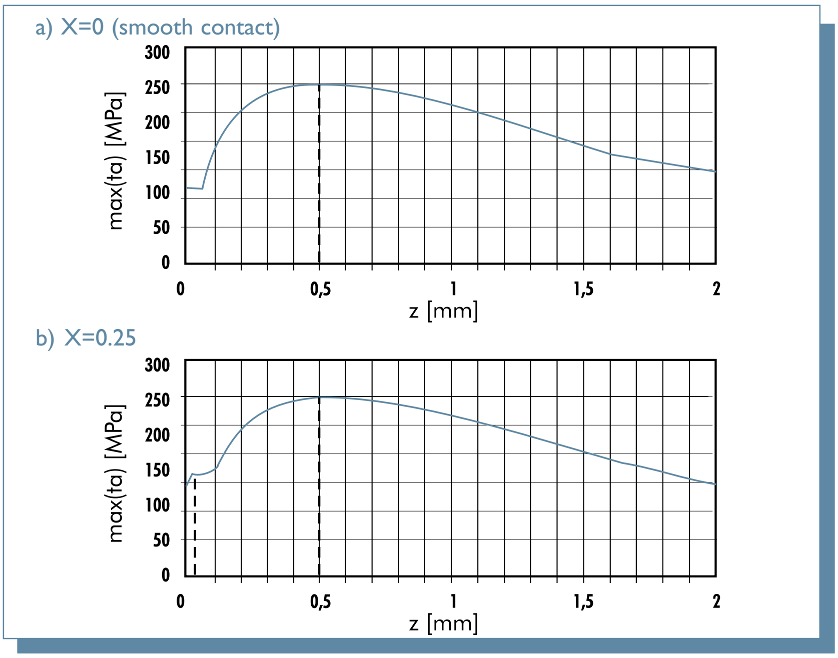


Fig.8: Evolution of damage with depth at the contact centre (p_{0eq}=1000 MPa)

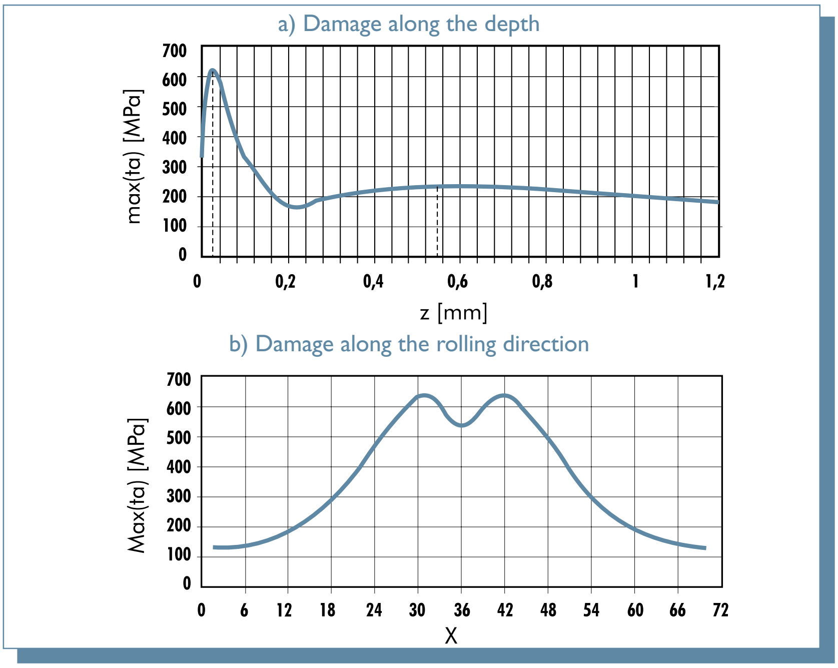


Fig.9: Damage evolution along x and z for a contact with X=4 and p_{0eq}=1000 Mpa

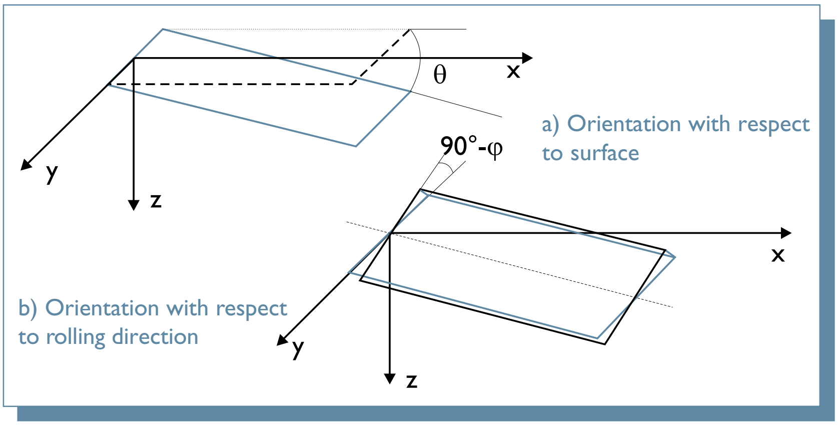


Fig.10: Orientation of a plane in the space

Zone II

	plane A	plane B	plane C	plane D	plane E	plane F
φ	90	90	180	180	180	145
θ	45	135	0	90	180	20
	plane G	plane H	plane I	plane L	plane M	
φ	145	145	145	145	145	
θ	25	30	150	155	160	

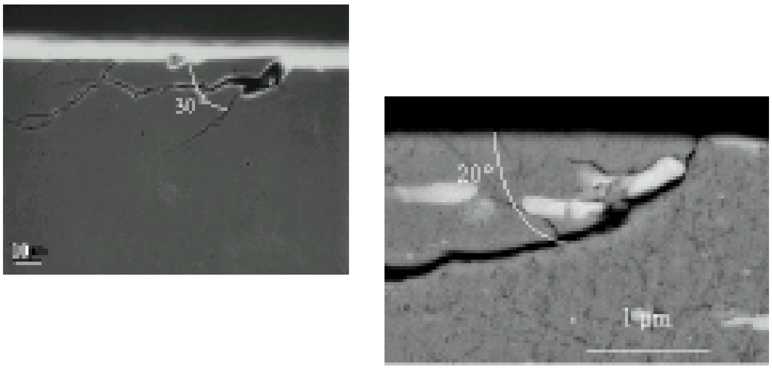


Fig.11: Directions of contact fatigue cracks in M50 steel [3]

CONCLUSIONS

A tool for the design of a rough contact charged with fatigue loads has been developed for this study. It is a curve that gives the tolerable limit pressure for a rough contact in function of the value of roughness, so that any fatigue damage appears. This pressure is calculated neglecting roughness, as the contact was smooth, it's then a hertzian pressure. This is the principal advantage of the tool proposed: the use of the curve needs only the application of the simple analytical formulas of Hertz and doesn't require calculations of the stress field in every point of the rough contact, for which a dedicated software would be necessary. The simplicity of use together with taking into account the most important phenomena in a rolling contact problem, make this curve a helpful tool in the contact design process.

For the curve establishment an existing multiaxial fatigue criterion, Dang Van criterion, has been modified to better adapt it to the specific conditions of a contact problem. The modification done allows to consider in a more realistic and conservative way dangers caused by mean compression stresses.

Finally it is important to remark that the results obtained have an experimental confirmation. The limit value of pressure of the smooth case is perfectly equivalent to the one found by Lamagnère and al. with a totally different approach and the critical directions of damage provided by calculations correspond to those observed experimentally by Dumont.

The results here presented must be read as an attempt to establish design tools for a mechanical contact and they represent then an effort to address researches of contact mechanics towards the design area. The authors are persuaded that this is the direction to follow in tribology studies in the next years: orientation to design needs.

ing. The results obtained reveal also the possibility of damage starting from planes C, D and E, perpendiculars to rolling direction and parallels or perpendiculars to surface. Any experimental confirmation exists at the moment for these types of crack nucleation.

For medium values of roughness (zone II) the same critical planes as before are found until X»4 and, in addition, planes with an orientation from 20° to 30° with respect to surface and of 55° with respect to rolling direction. Even in these cases an experimental confirmation of Dumont exists. Fig.11 shows the typical directions of contact fatigue cracks, as they can be observed experimentally; they have an orientation of about 20°-30° with respect to surface.

REFERENCES

1. DANG VAN, K. –*Sur la résistance à la fatigue des métaux* – Sciences et Techniques de l'Armement, Mémorial de l'artillerie française, Imprimerie nationale, Paris, vol. 47, 3éme fasc., 1973.
2. DUMONT, M.L., NELIAS, D., FLAMAND, L. – *Influence des rugosités et du frottement sur le tenseur des contraintes dans un massif élastique semi-infini* – Revue Française de Mécanique, vol. 1994-2, 1994, pp. 103-113.
3. DUMONT, M.L. – *Etude des endommagements de surface induits par fatigue de roulement dans les contacts élastohydrodynamiques pour des aciers M50 et 100Cr6* – Thèse Doct. Ing., Inst. Nat. Des Sciences Appliquées, Lyon, 1997, 197 p.
4. HERTZ, H. - *Uber die Berührung fester elasticher Körper* - J. Reine und angewandte Mathematik, 92, 1882, pp. 156-171.
5. JOHNSON, K.L. – *Contact mechanics* – Cambridge University Press, Cambridge, 1985, 452 p.
6. KENMEUGNE, B. – *Contribution à la modelisation du comportement en fatigue sous sollicitations multiaxiales d'amplitude variable* – Thèse Doct. Ing., Inst. Nat. des Sciences Appliquées, Lyon, 1996, 286 p.
7. LAMAGNERE, P., FOUGERES, R., LORMAND, G., VINCENT, A., GIRODIN, D., DUDRAGNE, G., VERGNE, F. – *A physically based model for endurance limit of bearing steels* – J. of Tribology, vol. 120, 1998, pp. 421-426.