Plastic Strain Energy in Low-Cycle Fatigue

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Abstract

Notched specimens, fabricated from a low alloy pressure vessel steel (A-533 B), were subjected to fatigue tests. These tests showed the influence of the theoretical stress concentration factor, Kt, on the fatigue life. A energy-based criterion has been adopted and it is shown that the total strain energy per cycle, $\Delta W_t = \Delta W_p + \Delta W_e$, is a proper damage parameter which can be used for the life prediction of notched elements.

Finite element analysis was carried out considering both the cyclic and monotonic stress-strain curves of the material. The strain energy was calculated and compared with the experimental values; these showed a good agreement and a life prediction was made.

Riassunto

Sono state eseguite delle prove sperimentali di fatica su dei provini intagliati (tipo keyhole). Per riuscire a valutare l'effetto dell'intaglio sulla fatica a basso numero di cicli sono stati considerati gruppi di provini con differenti raggi al fondo dell'intaglio, e quindi differenti coefficenti di sovrasollecitazione teorici K_t. Si è verificato che utilizzando un criterio di tipo energetico, in particolare basato sul lavoro di deformazione totale $\Delta W_t = \Delta W_e + \Delta W_p$ si riesce a valutare la vita a fatica di questi elementi intagliati.

Sono stati realizzati dei modelli numerici ad elementi finiti dei provini e introducendo sia la curva monotona del materiale, per riprodurre il primo ciclo di carico, sia quella ciclica è stato simulato il ciclo di isteresi e sono stati così calcolati i parametri energetici.

Introduction

The components of engineering structures are often subjected to variable loads. Geometrical discontinuities are generally the cause of stress increases which could eventually lead to fatigue failure if during initial loading the material yields locally and each loading cycle produces a stress-strain hysteresis loop. Fatigue life prediction for notched members may be approached from different viewpoints: the local strain approach and the energy criteria. The local strain approach [1],[2] considers that fatigue life is controlled primarily by the notch surface strain. Thus the fatigue data used are in the form of strain versus nucleation cycles curve, obtained from tests on smooth, axially loaded, specimens. To use a strain-life curve if the component contains a notch it is necessary to measure or to calculate the strain occurring locally at the notch. Estimation of the local strain is a very difficult process and it is possible to either consider the Neuber's rule [3] or to model the stress-strain behaviour [4],[5]. However even if a stress-strain history is determined a fatigue life prediction is not immediate. In fact experimental tests, conducted on keyhole specimens, have demonstrated that the theoretical stress concentration factor influences fatigue life [6],[7] and if the strain value calculated at the notch is introduced in to the strain-life curves, the corresponding fatigue life value is too conservative. The energy-based criterion [8] considers the definition of the plastic strain energy (see fig.l), i.e. the energy dissipated per cycle during the fatigue process. It is possible to correlate the fatigue damage within the material to the amount of plastic strain energy, which is defined as follows:

$$\Delta W_p = \int \sigma \cdot d\varepsilon_p \qquad (1)$$

Although the plastic strain energy per cycle is related to the low-cycle fatigue with a good agreement, there is nonetheless a limitation regarding its range of applicability since as so will. To predict long life failure the total strain energy per cycle, as a damage parameter, has been considered [9]:

$$\Delta W_t = \Delta W_p + \Delta W_e \qquad (2)$$

The fatigue life predictions given by means of the total strain energy definition show a good agreement with the experimental results, obtained from keyhole specimens [10]. The energy-based criterion seems to give good results even if a notch is present. In order to utilize this parameter it is, however, necessary to know the stabilized hysteresis loop of the material. The objective of this paper is therefore to present a numerical calculation of the total strain energy obtained by means of finite element program and to compare it with the experimental results.

Experimental Procedure

Stock material for the specimens consisted of 50 mm thick A-533 B steel plate. The smooth specimens were subjected to fully reversed strain-controlled cycles with a sinusoidally varying axial strain. Mechanical and cyclic properties are shown in Table 1.

TABLE 1 - Mechanical and cyclic properties of A-533 B steel

$R_{p0,2}$ [MPa]	R_m [MPa]	E [MPa]	K' [MPa]	n'	σ'_f [MPa]	ε'_f	b	c
566	682	210700	725	0,100	674	0,0991	-0,0532	-0,4472

Keyhole specimens with different notch radiuses and consequently different theoretical stress concentration factors K_t were used. Electrical strain gauges were attached to the specimens in the nucleation area and on the sides and back of the specimens in order to measure the strain values during the first loading cycle and the fatigue tests.
The load applied is pulsating from 0 to a maximum value P. At this value the yield point of the material is exceeded and, after the first load cycle has been applied, the average stress becomes almost 0, thus creating test conditions similar to those of strain-controlled tests.
All tests were terminated when a crack was detected by means of penetrating liquids.

Hysteresis loop

Uniaxial specimens

Assuming that the shape of the stress-strain path of the hysteresis loop is similar to the shape of the cyclic stress-strain curve, amplified by a factor of two (Masing's hypothesis), the plastic strain energy is expressed for a monoaxial load:

$$\Delta W_p = 4 \cdot \sigma_a \cdot \varepsilon_{a,p} \cdot \frac{(1 - n')}{(1 + n')} \tag{3}$$

or:

$$\Delta W_p = K \cdot \frac{(1 - n')}{(1 + n')} \cdot (2\,\sigma_a)^{\frac{1+n'}{n'}} \tag{4}$$

where

$$K = \frac{2}{(2\,K')^{\frac{1}{n'}}} \tag{5}$$

The plastic strain energy has been calculated for the smooth specimens using eq.(3) and introducing experimentally measured σ_a and ε_a values.

Eq.(4) has been also considered by introducing the experimental σ_a values.

Table 2 shows the plastic strain energy values determined experimentally and the ones calculated with (3) and (4). Agreement is good.

TABLE 2 - Plastic strain energy for unnotched specimens

Specimen	N_f	$\varepsilon_{a,p}$ [μm/m]	$\varepsilon_{a,t}$ [μm/m]	σ_a [MPa]	ΔW_pexp. [MJ/m^3]	ΔW_p (3) [MJ/m^3]	ΔW_p (4) [MJ/m^3]
P2	40	12718	15000	468,7	23,00	19,51	19,56
P7	44	10573	12000	446,9	17,52	15,46	11,58
P3	184	7725	10000	452,3	12,36	11,43	13,22
P1	1230	5139	7500	421,6	7,04	7,09	6,10
P4	1722	2845	5000	396,4	3,20	3,69	3,10
P6	8500	1139	3000	370,0	1,24	1,38	1,45
P8	18000	689	2500	355,5	0,75	0,80	0,93

Keyhole specimens

The hysteresis loops of the notched specimens are different from the ones of the smooth specimens. In fig.2 the stabilized cycles of the smooth specimen and the notched ones are compared in a diagram σ-ε: the sha

TABLE 3 - Tests results for keyhole specimens

Specimen	N_f	$\varepsilon_{a,p}$ [μm/m]	$\varepsilon_{a,t}$ [μm/m]	ΔW_pexp. [MJ/m^3]	ΔW_p (3) [MJ/m^3]	ΔW_p (4) [MJ/m^3]
2R-1	70000	115	2586	0,153	0,198	1,041
2R-2	50250	139	2647	0,189	0,255	1,327
2R-4	23000	330	3450	0,534	0,680	2,062
2Rl-1	85000	74	2466	0,065	0,127	0,921
2Rl-3	24500	187	2963	0,281	0,367	1,451
4R-1	61000	107	2451	0,164	0,184	0,892
4R-2	27000	183	2841	0,349	0,335	1,327
4R-3	13000	261	3140	0,422	0,513	1,682
4R-4	14600	337	3286	0,530	0,695	1,836
12R-0	21100	221	2697	0,312	0,358	1,141
12R-1	22000	268	2880	0,330	0,460	1,367
12R-2	20000	296	3031	0,422	0,542	1,540
12R-3	10000	419	3359	0,676	0,823	1,946
12R-4	7000	600	3925	1,185	1,237	2,664
12R-5	2150	1150	4900	2,063	2,578	3,922
12R-6	2500	1314	5449	2,737	3,172	4,736

pes are very different, even if the strain amplitudes, ε_a, of the two specimens are equal. The effective stress σ of the smooth specimen is obtained from the applied load by dividing it by the cross sectional area and it is equal to the nominal stress S. The nominal maximum stress S_{max} of the notched specimens is equal to the $K_t \cdot S$ value, where K_t is the elastic stress concentration factor. However it is important to note that if the $K_t \cdot S$ value exceeds the yield strength of the material, the nominal stress is no longer equal to the actual stress, σ, and more detailed analysis is required, in order to determine the local notch stresses and strains.
To calculate the actual σ-ε hysteresis loop area it is necessary to also know the local notch stresses-strains. In fact if eq.(4) is utilized, by introducing σ_a value determinated from the cyclic curve, the results are very different from the experimental values. Table 3 shows these results with the plastic strain energy values experimentally measured, calculated by means of eq.(4) and by means of σ_a, ε_a experimental values (eq.(3)).
To determine the plastic strain energy, when the σ and ε local values are not known, it is necessary to carry out an elastic-plastic stress analysis, using the finite element method.

Numerical Analysis

A threedimensional model of the keyhole specimen with notch radius R = 2 mm and $K_t = 3$ has been constructed. In fig.3 the model, which exploits the double simmetry, is reported; 20 node threedimensional isoparametric elements have been utilized with 3969 nodes and 11907 degrees of freedom.
Several elasto-plastic analyses have been carried out considering different stress-strain curves: both monotonic and cyclic ones. To compare the numerical values with the experimental ones curve $K_t \cdot S$-ε has been considered; in fact while the stresses calculated by means of finite element analysis are the actual stresses, the stresses determined by the experimental tests are the nominal stresses, K_t S. Therefore if the yield strength is exceeded, comparison of the σ-ε curves is impossible. In fig. 4 the $K_t \cdot S$-ε curves obtained by means of numerical calculation in the most stressed point of the specimen at the first loading cycle are compared with the experimental results. The values obtained from the analysis, which utilizes monotonic curve of the material, show a good agreement with experimental results, obtained during the first loading and unloading. On the other hand the use of a cyclic curve to describe the material mechanical behaviour allows us to calculate the plastic strain energy, which is compared with the stabilized hysteresis loop area measured during the fatigue tests. These finite element analysies, performed considering the cyclic stress-strain curve, also allow us to define an actual σ-ε stabilized cycle (see fig.5). Table 4 shows the hysteresis loop areas calculated by the finite element analysis, expressed both in terms of $K_t \cdot S$-ε and of σ-ε; it also shows these areas experimentally measured, expressed in terms of $K_t \cdot S$-ε.
The numerically determined plastic strain energy values, ΔW_p ($K_t \cdot S$-ε), are in good agreement with the experimental ones; the total strain energy has been calculated by adding the plastic strain energy numerical values to the elastic strain energy, ΔW_e:

$$\Delta W_e = \frac{1}{2E} \cdot \sigma_a^2$$

Fig.6 shows the ΔW_t-N_f curve, obtained from uniaxially loaded smooth specimens, to which the values obtained from the finite element analysis have been added. It is possible to see that there is a good agreement between the fatigue life predicted and the numerical values shown in Table 4.

Table 4 - **Experimental and numerical plastic strain energy for keyhole specimens**

Specimen	N_f [cycles]	Experimental $\Delta W_p(K_t \cdot S\text{-}\varepsilon)$ [MJ/m^3]	Numerical $\Delta W_p(K_t \cdot S\text{-}\varepsilon)$ [MJ/m^3]	Numerical $\Delta W_p(\sigma\text{-}\varepsilon)$ [MJ/m^3]
2R-1	70000	0,153	0,149	0,742
2R-2	50250	0,189	0,162	0,802
2R-4	23000	0,534	0,443	1,634

Conclusions

The problems of the fatigue life prediction of a notched element has been considered. An energy-based criterion was utilized and the predicted results are compared with the experimental data with a good agreement. In order to determine the total strain energy several finite element analyses were conducted and a comparison of the experimental and numerical hysteresis loops was made. The finite element analysis allows us to determine the actual σ-ε values of the notched specimens, otherwise not definable.

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Acknowledgments

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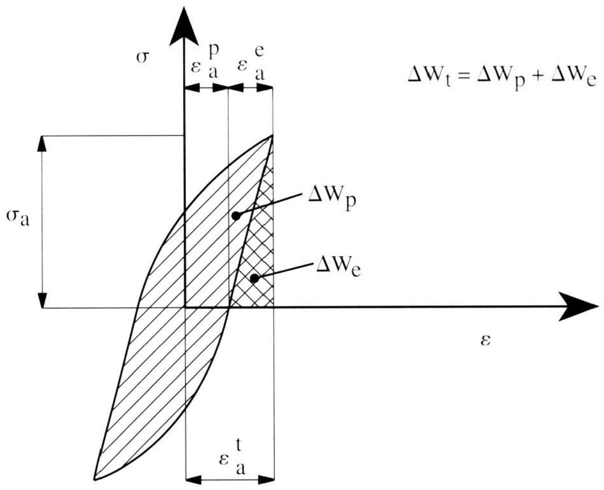

Fig. 1:
Strain energy definition

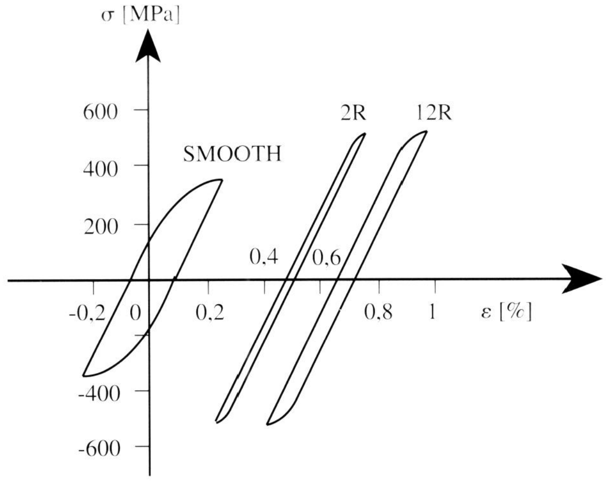

Fig. 2:
Hysteresis stabilized loops for smooth and keyhole specimens with R=2mm (2R) and R=12mm (12R)

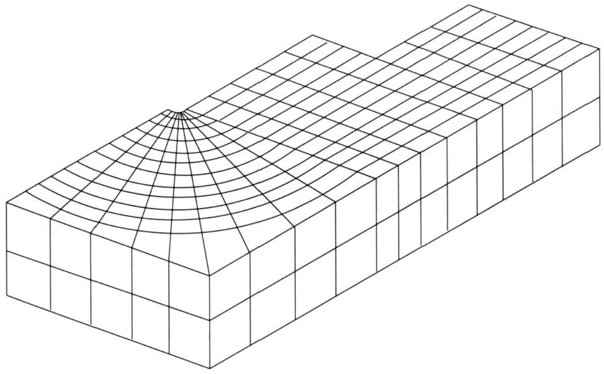

Fig. 3:
Finite element model of keyhole specimen with a radius R=2mm

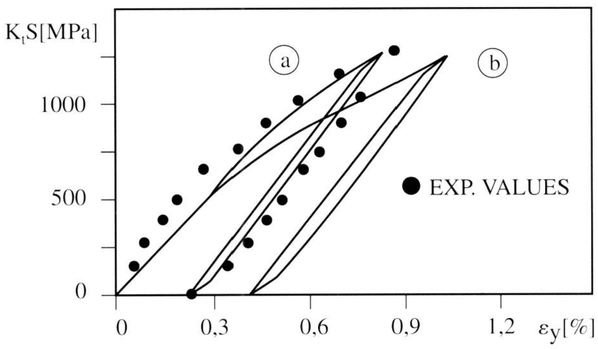

Fig. 4:
Numerical results: (a) is obtained introducing the monotonic curve of the material, (b) is obtained introducing the cyclic σ-ε curve of the material.
The values of (a) are comparable with experimental strain values measured during the first loading and unloading.

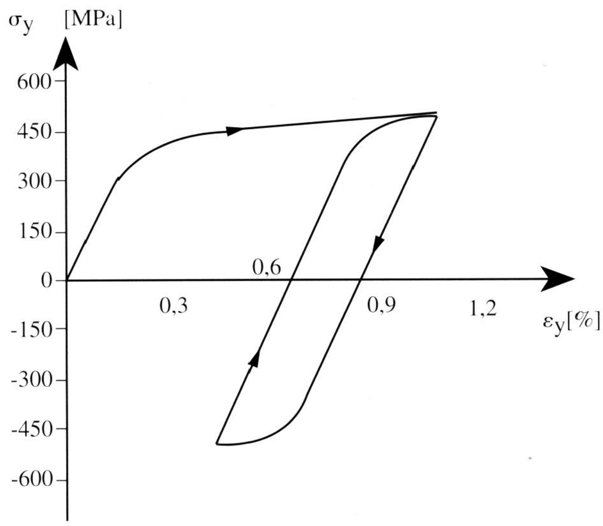

Fig. 5:
Numerically calculated actual σ-ε response of notched specimen

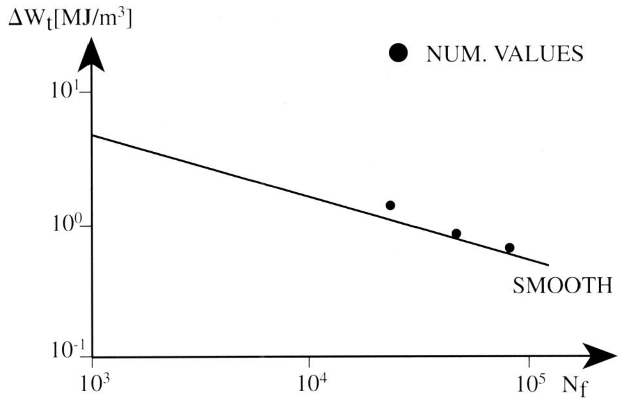

Fig. 6:
Comparison of smooth specimens and calculated data ΔW_t-N_f